RESEARCH EVIDENCE BASE

Math 180®

THE HMH RESEARCH MISSION STATEMENT

Houghton Mifflin Harcourt® (HMH®) is committed to developing innovative educational solutions and professional services that are grounded in learning science evidence and efficacy. We collaborate with school districts and third-party research organizations to conduct research that provides information to help improve educational outcomes for students, teachers, and leaders at the classroom, school, and district levels. We believe strongly in a mixed-methods approach to our research, an approach that provides meaningful and contextualized information and results.

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AN EQUATION THAT WORKS

With important contributions from the world's leading researchers and practitioners in mathematics education, Houghton Mifflin Harcourt has developed *Math 180* to provide a bridge for struggling students and their teachers. *Math 180* transforms math instruction so that students believe in the possibility of success and their teachers have cutting-edge tools to help them to meet the rigors of next generation math standards.

MANY STUDENTS ARE STRUGGLING TO ACHIEVE PROFICIENCY

59%

OF **4TH GRADERS**

&

66%

OF **8TH GRADERS** ARE ***AT BASIC*** OR ***BELOW BASIC*** IN MATH (NAEP, 2019)

MATH 180 STUDENTS REPRESENT OVER

IN OVER 900 DISTRICTS IN ALL 50 STATES

OVER 5,000

HMH MATH SERVICE DAYS

TEACHERS & EDUCATIONAL PROFESSIONALS ARE BENEFITING FROM OUR PROFESSIONAL *MATH SERVICE DAYS*

Math 180 AFFECTS TEACHER PRACTICE

88%

OF TEACHERS SHARED THAT ***Math 180* HAS IMPROVED** THEIR TEACHING OF MATHEMATICS

INTRODUCTION

To keep pace with a rapidly changing world, our nation's students must be prepared for the challenges they face after high school—in college, career, and citizenship. A strong mathematics preparation is essential to their future success. However, American students score below average on international tests of mathematical knowledge and skills (Loveless, 2011), and nearly two thirds of our nation's eighth graders do not meet current mathematics standards (U.S. Department of Education, 2019). Less than half of our college-bound students are prepared for post-secondary education and beyond (College Board, 2011). The nation's current equation for math education is not producing students ready to thrive in a 21st-century economy. The country needs a new equation that works.

A DEMAND FOR RIGOR—AN OPPORTUNITY FOR GROWTH

Across the country, states have adopted more rigorous standards for mathematics. Developed around a coherent progression of mathematical ideas and skills, the grade-by-grade objectives focus on core concepts that build on one another and incorporate mathematical thinking and habits of mind. These standards have come with new assessments that seek to provide a more accurate measure of student readiness for college and career. The demands of increased expectations provide an opportunity for innovations in curriculum to enhance and deepen teaching and learning (Conley, Drummond, de Gonzalez, Rooseboom, & Stout, 2011). The standards bring national attention to mathematics learning that is unprecedented, outlining what mathematics our students are to learn and ways in which they are to engage with content from kindergarten through high school.

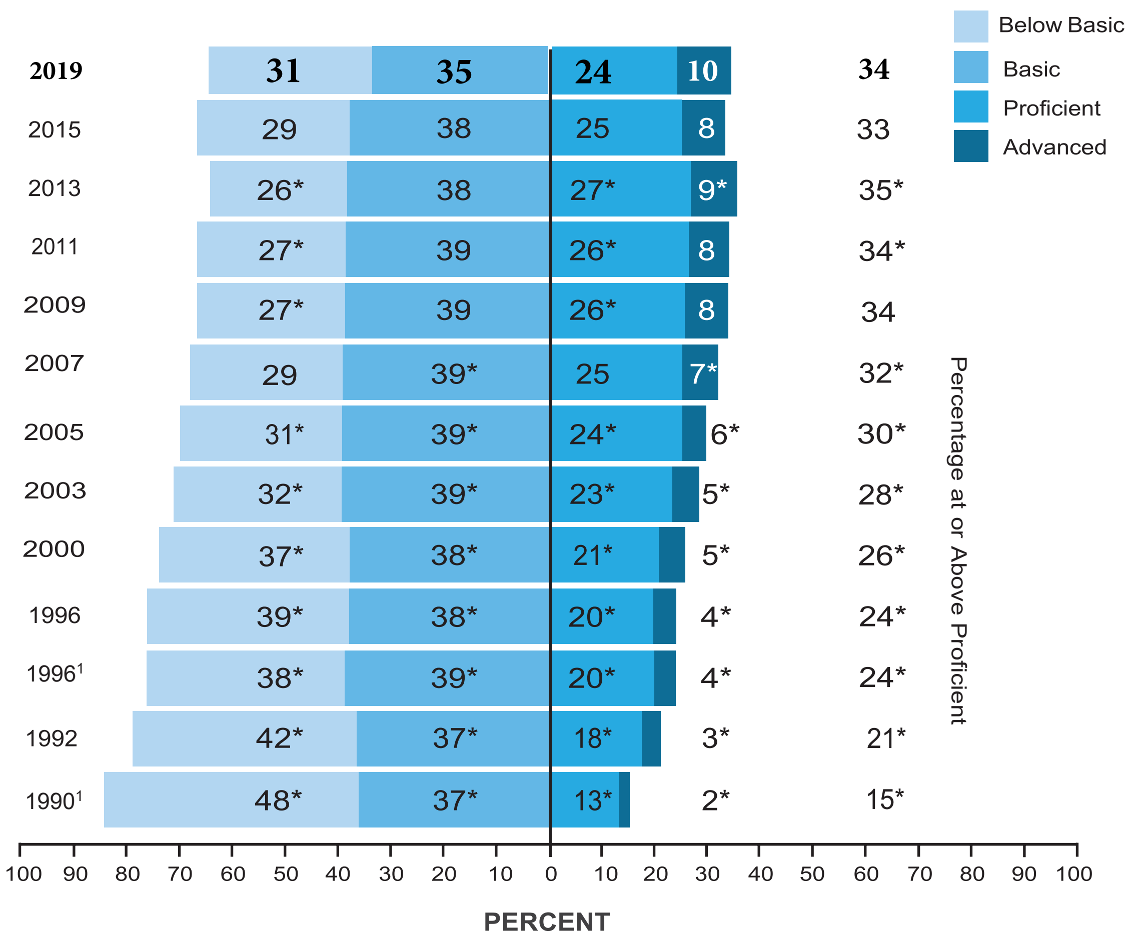
College- and career-readiness standards begin as early as Grade 6, when students are expected to solve problems involving ratios, rational numbers, and plane figures—key ideas in the development of algebraic knowledge and skills. Algebra is often identified as the gatekeeper to higher mathematics and thereby, a key to success in college and career (National Mathematics Advisory Panel [NMAP], 2008). Therefore, it is imperative that students have the mathematical foundation necessary for success to and through algebra. Indeed, this is an educational and social justice imperative.

In addition to providing a renewed perspective on the mathematics content students learn, these next generation standards require a focus on the act of doing mathematics and developing robust mathematical reasoning and thinking skills for all students. Mathematical knowledge and thinking skills have been identified as essential for college success (Conley, 2007).

Developing strong understandings of mathematics and keen abilities with the practices of mathematical thinking is particularly relevant for today's students, as jobs utilizing mathematical knowledge and skills are growing dramatically. The number of jobs in the fields of science, technology, engineering, and mathematics (STEM) is projected to increase at a rate of 8% in the next decade, which is more than double the growth of all other occupations (Zilberman & Ice, 2021).

As we look to improve students' mathematical preparedness for college and career, we must emphasize the role of the teacher in the implementation of new standards. The practices teachers employ have been learned and developed through participation in their own educational experiences as students (Ball, 1990; Cooney, 2001) and participation as teachers in communities of practice (Cobb & McClain, 2001). Additionally, teacher practices are filtered through what each teacher knows and believes. Therefore, teachers will implement the new standards upon negotiation of the objectives with their held knowledge and beliefs and current teaching practices (Cohen & Ball, 1990). Those designing effective support for teachers must consider this reality (Lappan, 1997) and the challenges teachers and their students face in adapting to these new expectations.

TREND IN EIGHTH-GRADE NAEP MATHEMATICS ACHIEVEMENT-LEVEL RESULTS



*Significantly different (p<.05) from 2015.
[1]Accomodations not permitted.
NOTE: Percentage-point differences are calculated based on the differences between unrounded percentages.

U.S. Department of Education Institute of Education Sciences; National Center for Education Statistics, National Department of Education Progress (NAEP), 2019 Mathematics Assessment.

MATH 180—REVOLUTIONARY MATH INTERVENTION

With the goal of ensuring that striving students become equipped with the knowledge, reasoning, and confidence to thrive in college and career, Houghton Mifflin Harcourt has developed a revolutionary mathematics program that motivates students to learn the content critical for success in algebra. Since the development of emotional and social competencies essential for success in college and career works hand in hand with efforts to improve students' content knowledge (Dweck, 2000; Yeager & Dweck, 2020), *Math 180* is rooted in relevant and realistic world connections, providing a rich landscape for learning in multiple domains. With a focus on the rigors of next generation math standards, *Math 180* leverages the research on effective mathematics teaching and learning and the need for educator support in implementing educational innovations to provide the support students need to develop key knowledge and skills essential for college and career success.

This report provides a detailed description of how *Math 180* utilizes the latest research to prepare students in Grades 5 and above with the mathematical understanding and skills they need to thrive in the 21st century. It summarizes the key research principles underlying the development of *Math 180* and delineates the specific program features that are designed to engage and empower learning experiences, support teachers in maximizing instructional effectiveness, and give administrators the tools to ensure high-quality implementation.

Only 34% of eighth graders are at or above proficiency in math.

OVERVIEW OF *MATH 180*

Three research-based principles have been engineered into *Math 180* to transform math instruction so that students believe in the possibility of success and their teachers have cutting-edge tools to accelerate their learning of the rigorous next generation math standards.

FOCUS ON WHAT MATTERS MOST

Accelerating Student Learning
For middle school students who are more than one year behind in math, reteaching every missed skill and concept simply isn't possible. *Math 180* focuses on rebuilding the essential concepts and skills that underlie later math learning along a progression to algebra. Carefully curated by expert mathematicians and next generation standards architects, the *Math 180* scope and sequence are built around a focused and coherent curriculum that enables struggling students to progress quickly and effectively toward grade-level curriculum.

FORCE MULTIPLIER FOR TEACHING

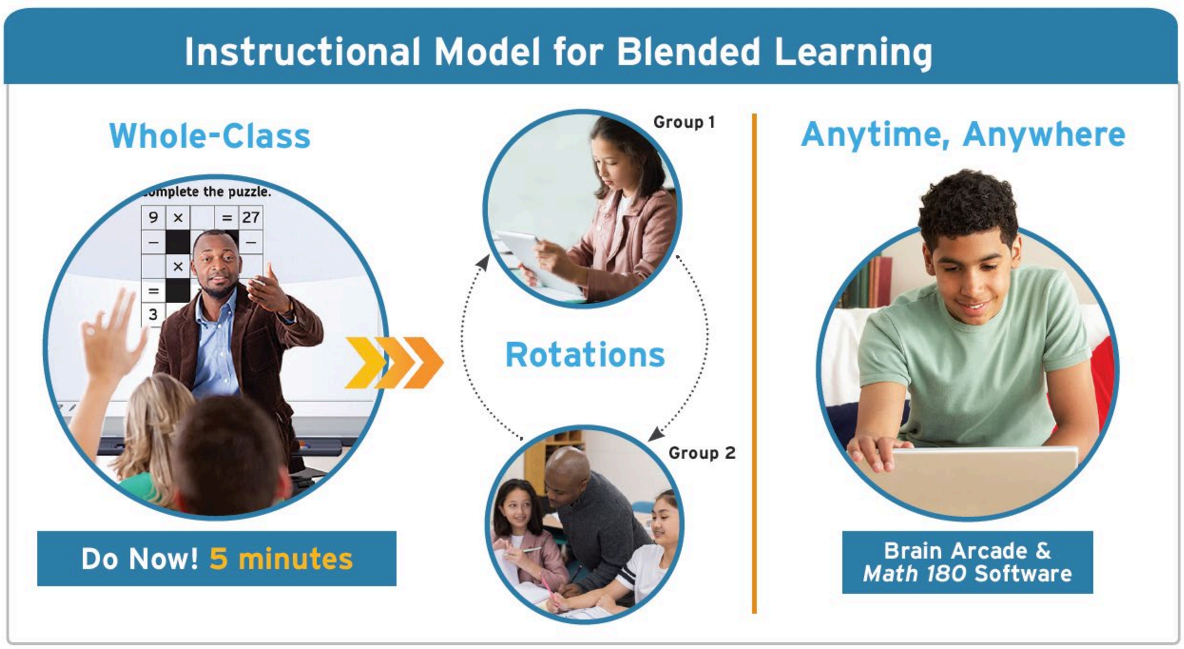
Build Teacher Effectiveness
A force multiplier is an approach that dramatically increases—or multiplies—effectiveness. Teachers are the key force behind effective math instruction, yet for most school districts, teacher preparedness has become a critical issue. *Math 180*'s professional learning scaffolds less experienced teachers and provides a wealth of sophisticated supports to veteran math teachers. Guided by Dr. Deborah Ball, the country's most respected voice in building teaching capacity, *Math 180* helps teachers become force multipliers by surrounding them with the resources they need to be greater at what they do best.

GROWTH MINDSET

Attitudes Toward Knowledge Can Impact Performance
Too many students and their teachers have come to believe that math success is impossible. This "fixed mindset" undermines effort and becomes self-fulfilling. *Math 180* incorporates the work of Dr. Carol Dweck's organization, Mindset Works®, in all aspects of the program to create a "growth mindset" culture in the classroom. Students learn that knowledge is malleable and reflect together on effective strategies for learning while building confidence and monitoring individual growth in an adaptive software environment.

MATH 180 INSTRUCTIONAL MODEL FOR BLENDED LEARNING

The flexible instructional model in *Math 180* maximizes instructional time with a clear organization for whole-class, group, and individualized learning. This simple yet powerful design allows both the teacher and technology to deliver efficient, effective instruction based on their respective strengths.



WHOLE-CLASS DO NOW

This classroom management routine develops mathematical thinking and makes connections to prior topics.

GROUP INSTRUCTION

The teacher facilitates instruction to build conceptual understanding, develop reasoning and communication skills, and interpret student thinking.

PERSONALIZED SOFTWARE

The *Math 180* software adapts to each student's needs, providing added practice for those who need it and accelerating those ready to move on.

BRAIN ARCADE

Available anytime, anywhere, the Brain Arcade provides each student with a personalized playlist of games that build strategic and procedural fluency.

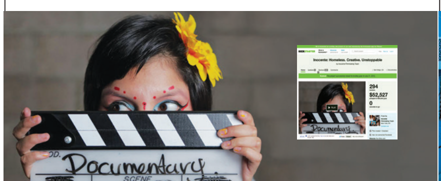
A COMPREHENSIVE SYSTEM TO RAISE MATH ACHIEVEMENT

Instructional Focus

Designed for students who lack numerical understanding and reasoning skills, the Block Series in *Math 180* focuses on rebuilding key concepts of numbers and operations that enable students to rebuild foundational skills using algebraic thinking to develop relationships between operations and with real numbers.

Multiplication and Division

BLOCK 1 — Multiplicative Thinking
Buzz Worthy



- TOPIC: Equal Groups in Multiplication
- TOPIC 2: Facts and Factors
- TOPIC 3: 10 as a Factor

CAREER CLUSTER: Marketing and Advertising

BLOCK 2 — The Distributive Property
Designing Your World

- TOPIC 1: Place Value in Multiplication
- TOPIC 2: Strategies for Multiplication
- TOPIC 3: Two-Digit Multiplication

CAREER CLUSTER: Art and Design

BLOCK 3 — Division
On a Mission

- TOPIC 1: Equal Groups In Division
- TOPIC 2: Strategies for Division
- TOPIC 3: Partial Quotient Strategy

CAREER CLUSTER: Community & Public Service

Fractions

BLOCK 1 — Fraction Concepts
Be My Guest

- TOPIC 1: Understanding Fractions
- TOPIC 2: Equivalent Fraction Models
- TOPIC 3: Fractions as Division

CAREER CLUSTER: Hospitality and Tourism

BLOCK 2 — Fraction Relationships
Doctor's Orders

- TOPIC 1: Strategies for Comparing Fractions
- TOPIC 2: Equivalent Fractions on a Number Line
- TOPIC 3: Adding and Subtracting Fractions

CAREER CLUSTER: Health and Medicine

BLOCK 3 — Fraction Multiplication and Division
Out of This World

- TOPIC 1: Unit Fraction Multiplication
- TOPIC 2: Strategies for Multiplying Fractions
- TOPIC 3: Strategies for Dividing Fractions

CAREER CLUSTER: Science and Engineering

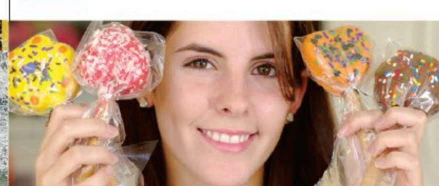
Decimals and Integers

BLOCK 1 — Decimals and Place Value
How Does It Work

- TOPIC 1: Fraction and Decimal Relationships
- TOPIC 2: Decimal Place Value
- TOPIC 3: Decimals on a Number Line

CAREER CLUSTER: Computers and Technology

BLOCK 2 — Decimal Operations
You're the Boss



- TOPIC 1: Adding and Subtracting Decimals
- TOPIC 2: Decimal Multiplication
- TOPIC 3: Patterns in Decimal Division

CAREER CLUSTER: Business and Management

BLOCK 3 — Both Sides of Zero
Final Frontier

- TOPIC 1: Rational Numbers on a Number Line
- TOPIC 2: Adding Rational Numbers
- TOPIC 3: Subtracting Rational Numbers

CAREER CLUSTER: Environmental Science

Math 180 picks up the progression to algebra at whole number multiplication and builds a coherent narrative of understanding through fractions and decimals, proportional reasoning, and functional thinking. Concepts, visual models, and procedural strategies build on one another, opening up and facilitating new learning. Students enter the progression at different points based on their existing knowledge and progress at their own pace. Instruction is organized into six Block Series, each with three blocks of instruction, featuring high-interest career themes. The focused content helps students make connections while learning to think algebraically.

Instructional Focus

Intended for students with foundational skills of numbers and operations, these Block Series build and focus on proportional, linear, and functional relationships.

Rates and Ratios

BLOCK 1 — Rates in Time
Moving Forward

TOPIC 1: Distance-Time Graphs
TOPIC 2: Representing Rates
TOPIC 3: Comparing Rates
CAREER CLUSTER: Logistics

BLOCK 2 — Rate and Ratio Concepts
Bright Future

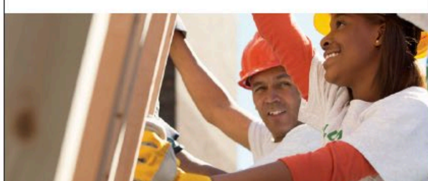
TOPIC 1: Comparing Quantities
TOPIC 2: Ratio Concepts
TOPIC 3: Equivalent Ratios
CAREER CLUSTER: Environmental Science

BLOCK 3 — Ratio Relationships
The Bottom Line

TOPIC 1: Representing Ratios
TOPIC 2: Applications of Ratio Understanding
TOPIC 3: Equivalent Ratios
CAREER CLUSTER: Sales & Marketing

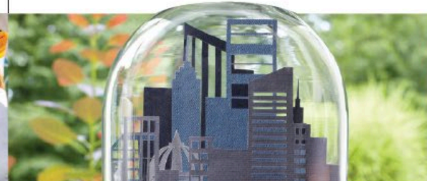
Proportional and Linear Relationships

BLOCK 1 — Percent and Proportional Reasoning
For the People



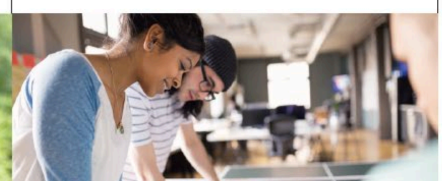
TOPIC 1: Percent Concepts
TOPIC 2: Comparing with Percent
TOPIC 3: Relationships In Ratio Tables
CAREER CLUSTER: Public Services

BLOCK 2 — Proportional Relationships
Imagine That



TOPIC 1: Representing Proportional Relationships
TOPIC 2: Solution Sets
TOPIC 3: Applications of Percent
CAREER CLUSTER: Art & Design

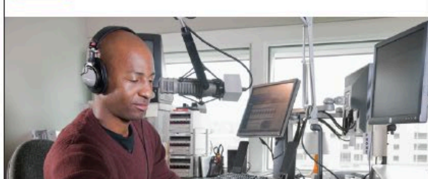
BLOCK 3 — Linear Relationships
On the Money



TOPIC 1: Linear Equations
TOPIC 2: Slope of a Line
TOPIC 3: Interpreting Linear Equations
CAREER CLUSTER: Entrepreneurship & Business

Linear and Nonlinear Functions

BLOCK 1 — Graphs in the Plane
Make Yourself Heard



TOPIC 1: Patterns with Negative Numbers
TOPIC 2: Negative Slope
TOPIC 3: Operations With Negative Numbers
CAREER CLUSTER: Entertainment

BLOCK 2 — Functions
Crack the Code

TOPIC 1: Representing Functions
TOPIC 2: Analyzing Functions
TOPIC 3: Squares and Square Roots
CAREER CLUSTER: Information Technology

BLOCK 3 — Systems of Equations
Take Care

TOPIC 1: Comparing Linear Functions
TOPIC 2: Reasoning With Linear Systems
TOPIC 3: Solutions of Systems
CAREER CLUSTER: Health Science

MATH 180 ADVISORS

Math 180 is the result of Houghton Mifflin Harcourt's collaboration with the leading math education researchers, thinkers, and practitioners.

DR. DEBORAH BALL

Dr. Deborah Ball, the Dean of the School of Education at the University of Michigan, is one of the nation's foremost voices on building teacher effectiveness and the founder of TeachingWorks—an organization that aims to transform how teachers are prepared and supported. Serving as the lead advisor on teacher-facilitated instruction, Dr. Ball has helped to embed TeachingWorks' research-based High Leverage Teaching Practices into every *Math 180* lesson.

DR. TED HASSELBRING

Dr. Ted Hasselbring is a Professor of Special Education at Peabody College of Vanderbilt University and author of *READ 180*® and *System 44*®. Dr. Hasselbring researched the use of technology for enhancing learning in students with mild disabilities and those who are at risk of school failure. With *Math 180*, Dr. Hasselbring shares his expertise in adaptive technology to build a personalized learning experience for struggling math students.

HAROLD ASTURIAS

Harold Asturias is the Director of the Center for Mathematics Excellence and Equity at the University of California, Berkeley, where he designed and implemented professional development for K–12 math teachers who teach multilingual learners. Asturias has integrated explicit language goals and vocabulary routines as well as support for multilingual learners into *Math 180*.

DR. FREEMAN HRABOWSKI

Dr. Freeman Hrabowski is the President of the University of Maryland, Baltimore County, where he has received national recognition and publicity for his work to promote math and science to minority students. He was appointed by President Obama as the Chair of the National Education Commission and has contributed his guidance to help *Math 180* deliver on the promise to engage struggling learners and foster better family involvement.

DR. JON STAR

Dr. Jon Star is an educational psychologist who studies children's learning of mathematics in middle and high school, particularly algebra. Star's current research explores the development of flexibility in mathematical problem solving. Star also investigates instructional and curricular interventions that may promote the development of mathematical understanding. Star's most recent work is supported by the National Science Foundation. In addition, Star is interested in the preservice preparation of middle and secondary mathematics teachers. Prior to his graduate studies, Star spent six years teaching middle and high school mathematics

MINDSET WORKS®

Mindset Works is a social venture that helps human beings realize their full potential. Founded by leading motivational researcher Dr. Carol Dweck and her colleague Dr. Lisa S. Blackwell, the organization focuses on translating lessons into programs that schools can use to increase student motivation. Mindset Works has been influential in driving *Math 180*'s transformative growth mindset principle.

MATH SOLUTIONS®

Math Solutions is the nation's leading provider of math professional development. Math Solutions was instrumental in the program-embedded Professional Learning, which gives teachers content and pedagogic background for every topic.

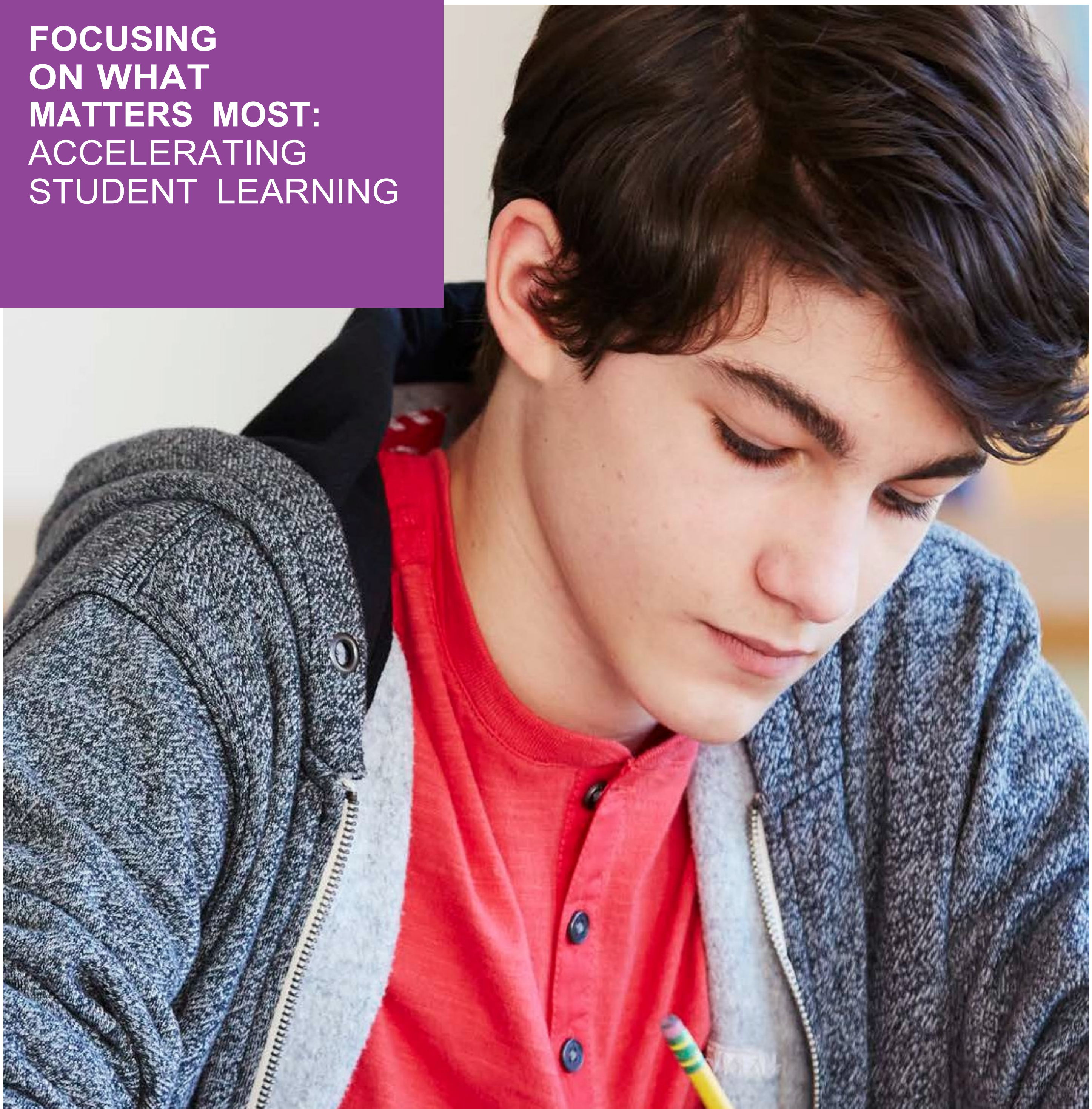
CENTER FOR APPLIED SPECIAL TECHNOLOGY (CAST)

Founded in 1984 as the Center for Applied Special Technology, CAST is a nonprofit research and development organization that works to expand learning opportunities for all individuals, especially those with disabilities, through Universal Design for Learning.

SRI EDUCATION

SRI Education is a division of SRI International, one of the leading research and development organizations in the world. The SRI team helped to identify key research and advisors to guide *Math 180* development and in the implementation of *Math 180*'s dynamic visual models.

FOCUSING ON WHAT MATTERS MOST: ACCELERATING STUDENT LEARNING



Carefully curated by leading thinkers and practitioners, the ***Math 180*** scope and sequence are built around a focused and coherent curriculum that enables struggling students to make connections while learning to think algebraically. ***Math 180*** teaching practices focus on building conceptual understanding with a capacity for disciplined reasoning, analysis, argument, and critique.

***Math 180* focuses on mathematical thinking and the specific set of concepts along the progression to algebra.** Research shows that to accelerate learning, students and teachers must be both effective and efficient, developing deep conceptual understanding and then persistently activating and building on prior knowledge to foster new neural connections. Striving math students often lack the explicit mathematical vocabulary they need to understand the problems they are expected to solve or to communicate their lack of understanding. *Math 180* considers an understanding of the language of mathematics across all subject areas to be a vital part of the progression to algebra. Similarly, the Standards for Mathematical Thinking provide an affective-behavioral profile of proficient mathematical learners.

Our approach relies on four foundational principles:

1. **Focus:** Concentration on the concepts along the progression to algebra
2. **Coherence:** Emphasis on the interdependence and cumulative nature of mathematics
3. **Rigor:** Robust opportunities for high-order thinking and reasoning in routine and nonroutine problems
4. **Communication and Standards for Mathematical Thinking:** Explicit mathematical vocabulary instruction, English language support, and active development of the behaviors employed by proficient mathematics learners

FOCUS ON WHAT MATTERS MOST

Math 180 **ensures that older students who have not been successful now have an explicit, accelerated path to algebra.** This learning progression is divided into six Block Series. Students who are significantly below grade level have not developed the numerical understanding and fluency needed to make sense of more complex math. Building these critical foundations is the goal of the earlier Block Series. Students whose difficulties lie with abstract pre-algebra concepts will benefit from the later Block Series' instructional focus on developing proportional reasoning.

PROGRESS TO ALGEBRA IN GRADES K–8*

K	1	2	3	4
Know number names and the sequence	Represent and solve problems involving addition and subtraction	Represent and solve problems involving addition and subtraction	Represent and solve problems involving multiplication and division	Use the four operations to solve problems
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Add and subtract within 20	Understand properties of multiplication and the relationship to division	Generalize place value understanding for multi-digit numbers
Compare numbers	Add and subtract within 20	Understand place value	Multiply and divide within 100	Use place value understanding and properties to perform multi-digit arithmetic
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Work with addition and subtraction equations	Use place value understanding and properties of operations to add and subtract	Solve problems involving the four operations, and identify and explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering
Work with numbers 11–19 to gain foundations for place value	Extend the counting sequence	Measure and estimate lengths in standard units	Develop understanding of fractions as numbers	Build fractions from unit fractions
	Understand place value	Relate addition and subtraction to length	Solve problems involving measurement and estimation of time, volume, and mass	Understand decimal notation for fractions, and compare decimal fractions
	Use place value understanding and properties of operations to add and subtract		Understand concepts of area, and relate area to multiplication and to addition	
	Measure lengths indirectly and by iterating length units			

Pre-requisite

Multiplication and Division
Fractions
Decimals and Integers

Rates and Ratios
Linear and Proportional Reasoning
Linear and Nonlinear Functions

Middle school concepts are focused on applying and extending prior understandings, those that should have been established in earlier grades. Students need to rebuild these foundations in order to move forward and accelerate.

5

Understand the place value system

Perform operations with multi-digit whole numbers and decimals to the hundredths

Use equivalent fractions as a strategy to add and subtract fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions

Understand concepts of volume, and relate volume to multiplication and to addition

Graph points in the coordinate plane to solve real-world and mathematical problems

6

Apply and extend previous understandings of multiplication and division to divide fractions by fractions

Apply and extend previous understandings of numbers to the system of rational numbers

Understand ratio concepts, and use ratio reasoning to solve problems

Apply and extend previous understandings of arithmetic to algebraic expressions

Reason about and solve one-variable equations and inequalities

Represent and analyze quantitative relationships between dependent and independent variables

7

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers

Analyze proportional relationships, and use them to solve real-world and mathematical problems

Use properties of operations to generate equivalent expressions

Solve real-life and mathematical problems using numerical and algebraic expressions and equations

8

Work with radical and integer exponents

Understand the connections between proportional relationships, lines, and linear equations

Analyze and solve linear equations and pairs of simultaneous linear equations

Define, evaluate, and compare functions

Use functions to model relationships between quantities

* *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics (2012).*

FOCUS

To accelerate learning for students below grade level, *Math 180* maintains a tight focus on the concepts, strategies, and content knowledge that matter: those that constitute the progression to algebra. Guided by the world's leading mathematics researchers, thinkers, and practitioners, *Math 180* delivers a concise and highly focused mathematics framework designed to maximize instructional time.

RESEARCH AND EXPERT OPINION

- Given the amount of math content that is available choices must often be made in terms of what students should focus on and the benefits and drawbacks for those decisions (Li & Schoenfeld, 2019).

- When states in the United States adapted their curriculum to have fewer, more focused standards, similar to Singapore, their NAEP scores improved significantly (Ginsburg, Leinwand, Anstrom, & Pollock, 2005).

- Algebra teachers have identified that students with deficiencies in whole number arithmetic, fractions, ratios, and proportions struggle with algebra (NMAP, 2008).

- Mathematics intervention for students in Grades 4–8 should focus on rational numbers and whole number arithmetic. Covering fewer topics in more depth is particularly important for students who struggle with mathematics (Gersten et al., 2009).

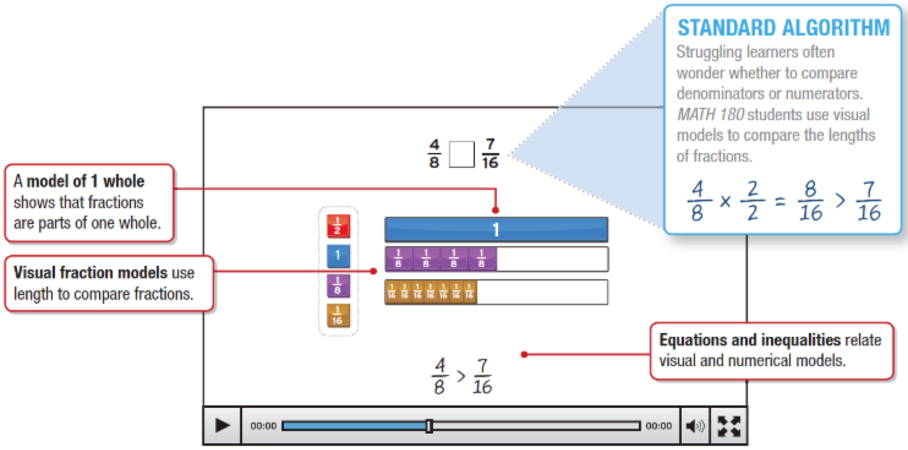
- Development of division and fraction mastery has been shown to forecast later mathematical proficiency, including student performance in algebra and more advanced mathematics courses (Siegler et al., 2012).

- To attain mathematical proficiency, students need to develop problem-solving skills, methods for using their knowledge effectively, and positive dispositions toward mathematics, along with content knowledge (Schoenfeld, 2007).

- Content that develops mathematical proficiency should grow in complexity, be engaging, and develop important mathematical ideas at appropriate depth (Conley, 2011; NMAP, 2008).

- “Applications and modelling play a vital role in the development of mathematical understanding and competencies” (Ministry of Education Singapore, 2006, p. 4). A key feature of the Singapore national mathematics curriculum is the use of the model method—providing students with concrete, pictorial, and abstract approaches to problem solving.

- Students often struggle with the transition from arithmetic to algebra (National Research Council (NRC), 2001), which may be a result of weak number and operation knowledge (ACT, 2010), including a deficit in student rational number understanding (Kloosterman, 2010; NCTM, 2007; Siegler et al., 2010), and limited ability to solve contextually based problems (Hoffer, Venkataraman, Hedberg, & Shagle, 2007).



Learn the *Math 180* Strategy— example Fractions.

HOW *MATH 180* DELIVERS

Math 180 accelerates students to algebra by providing a concise, logical curriculum tightly focused on building deep conceptual understanding and fluency. Each lesson provides a new concept or strategy that will be used again and again in future lessons, as students continually expand their toolbox of mathematical vocabulary terms, procedural strategies, and interactive virtual manipulatives. Division strategies are extended when students study fractions, and fraction strategies are extended when students begin to understand the relationship between decimals and fractions.

Math 180 focuses on the progression to algebra as well as the Singapore national curriculum, which emphasizes the use of visual models and a strong, transparent conceptual progression. Within each lesson of *Math 180*, interactive mTools— such as the bar model, fraction strips, and the motion model— are used in conjunction with novel problem-solving contexts to demonstrate the applications of mathematical strategies. In the Learn Zone, students extend their new strategies and models with the proper scaffolds, gradually moving from guided to independent practice with the aid of a metacognitive coach.

New standards state that the application and extension of foundational mathematics concepts form the core of algebra instruction: Rational expressions are an extension of rational numbers, while polynomials are a natural extension of integers. Students in *Math 180* are explicitly shown how to make these connections, continually applying and extending their understanding of arithmetic operations, rational numbers, and proportional reasoning into new contexts. The student software ensures that students make such connections actively, and the classroom routines, guided practice, and activities in the student *mSpace* ensure that students are extending their previous understandings consciously and logically.

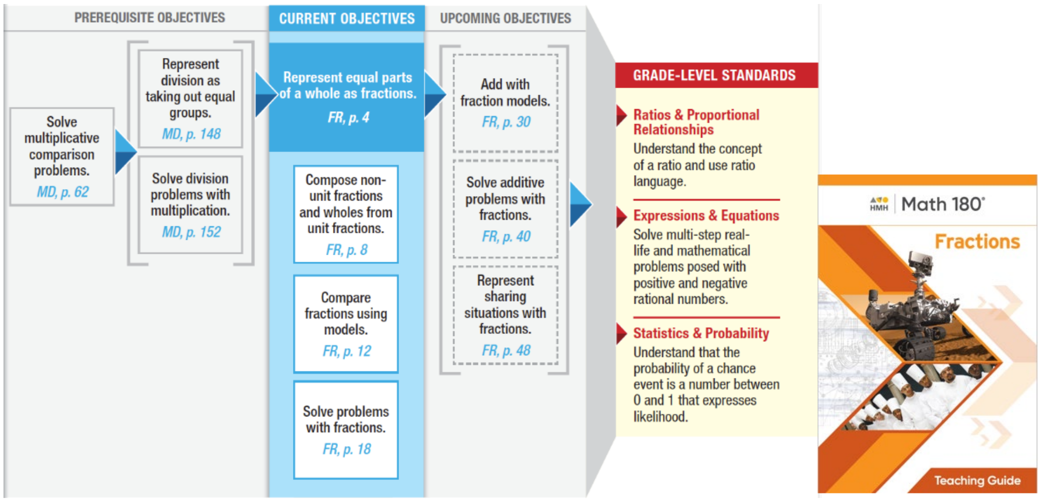
Students in *Math 180* are explicitly shown how to make these connections, continually applying and extending their understanding of arithmetic operations, rational numbers, and proportional reasoning into new contexts.

COHERENCE

An essential element of *Math 180* is the delivery of a coherent curriculum in which concepts build on one another cumulatively and in a logical progression. Rather than being an unlinked series of tricks to solutions, math should make sense. Knowledge should build as progressions between grades and between topics (Core Standards, 2012). Students in *Math 180* progress from concrete to pictorial to abstract representations of each concept. In each successive unit of *Math 180*, students are encouraged to activate prior learning and access the models and strategies common to multiple topics. This way, students build mental connections between topics and transfer knowledge with a reduced strain on memory-retrieval processes.

RESEARCH AND EXPERT OPINION

- The goals of a coherent curriculum framework are to recognize learning as a developmental progression, focus on a fundamental set of key ideas and to integrate both knowledge and practice (Penuel & Reiser, 2018).
- Mathematics instruction must offer students a carefully sequenced balance of conceptual and skill-based instruction and adhere to important instructional principles, such as the Standards for Mathematical Thinking, for learning in mathematics (NCSM, 2013a). Technology should be used to support the transition from basic to more advanced understandings of mathematics concepts, and content should be spiraled to ensure understanding.
- "A+ countries," those countries that were the highest-achieving on TIMSS, build on early foundational skills year after year. It is this coherence that is one of the most important characteristics defining quality content standards (Schmidt, Houang, & Cogan, 2002).
- Instructional tasks that develop mathematical understanding build on students' previous learning and interests and cause students to struggle in meaningful ways to develop important mathematical ideas (Hiebert et al., 1997).
- External representations, such as models, help students understand abstract mathematics (Gersten et al., 2009). These models assist students in moving through a progression from concrete to pictorial to abstract representations of mathematical content (Wong, 2004). Instructional programs should enable all students to "select, apply, and translate among mathematical representations to solve problems" (National Council of Teachers of Mathematics (NCTM), 2000).
- Technology that affords students the opportunity to practice new skills systematically, with information presented in manageable sets, fosters automaticity, which reduces the strain on memory retrieval processes (Hasselbring & Goin, 2004).
- In conjunction with the Standards for Mathematical Thinking calling for the use of appropriate tools strategically, the National Council of Supervisors of Mathematics (NCSM, 2013b) recommends that teachers consistently use manipulatives to build conceptual understanding of mathematics. Manipulatives can help students build links between the object, the symbol, and the mathematical idea being represented (NRC, 2001) and are a useful strategy for helping students to build a foundation in mathematics, particularly for students who have previously struggled with mathematics (NCSM, 2013b).
- Manipulative materials help students make sense of abstract ideas, provide students with ways to test and verify ideas, are useful tools for solving problems, and make mathematics learning more engaging and interesting by lifting mathematics off textbook and workbook pages (Burns, 2007).



Professional Learning in the *Teaching Guide*

HOW *MATH 180* DELIVERS

Struggling students often fail to see the interrelatedness of subsequent math topics and cannot, on their own, bring prior knowledge to bear when learning new concepts. *Math 180* celebrates the coherence, interdependence, and cumulative nature of mathematics concepts through the mechanics of interactive learning and game mechanics: mastering a concept "unlocks" the successive concepts that build on it. *Math 180* also supports coherence in blended learning by introducing instructional models in a consistent manner across all learning environments. A student will use common strategies on the daily Do Now, in Group Instruction lessons, and in lessons on the student software.

Each block of instruction is designed to be internally cohesive and to unfold clearly and logically. Each concept builds on preceding concepts. During Group Instruction, students explicitly "connect" the new concept to prior learning, and on the student software, each student is reminded to apply and extend their interactive mTools—interactive virtual manipulatives—into new conceptual contexts. In accordance with the Singapore model, students gain understanding of abstract math by explicitly connecting each math concept across multiple modalities, including in context, pictures, symbols, and words.

Between blocks, *Math 180* uses consistent mathematical language to activate prior learning as new concepts are introduced. In the scope and sequence of the blocks, vital transitions—such as the transition from division to fractions or from proportional relationships to linear equations—are made explicit both by the virtual coaches and through the instructional model. Strands of instruction apply to multiple blocks of instruction. When students learn how to work with place value in whole number operations and again in decimal operations, the mathematical language and models are consistent to reduce cognitive overload and support transfer.

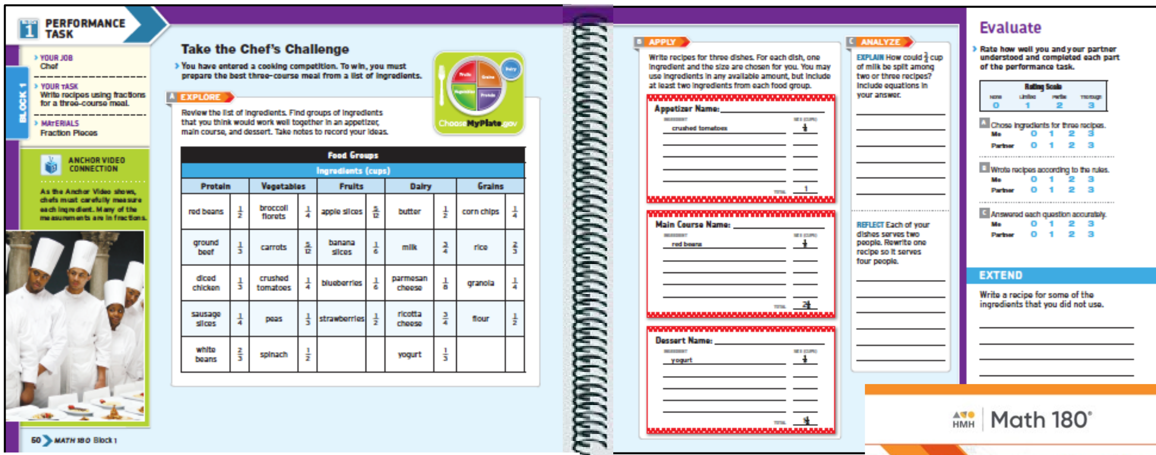
Each concept builds on preceding concepts; during Group Instruction, students explicitly "connect" the new concept to prior learning.

RIGOR

Rigor describes the degree to which sets of standards address key content that prepares students for success beyond high school (Achieve, 2010). To meet the expectations of a rigorous set of standards, *Math 180* delivers on three aspects of rigor: conceptual understanding, procedural skills and fluency, and applications. Rather than trying to teach students a little bit of everything, learning and understanding must be deeper, with high learning expectations for each student.

RESEARCH AND EXPERT OPINION

- The rigor present in classroom instruction should also be applied to remote learning opportunities. When instruction moves outside of the classroom, it is possible to adapt instructional frameworks to avoid compromising rigor (Blackburn & Miles, 2020).
- "Rigor is creating an environment in which each student is expected to learn at high levels, and each is supported so he or she can learn at a high level, and each student demonstrates learning at a high level" (Blackburn, 2008, p. 3).
- "The coherence and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra" (NMAP, 2008, p. 18).
- For instructional purposes, technology aids students in developing meaningful mathematics through the advancement of computational fluency (Hasselbring, Lott, & Zydney, 2006).
- Students' content knowledge improves as they solve problems with authentic contexts (Gersten et al., 2008), develop understanding of problem structures, and gain access to flexible solution strategies (Jitendra & Star, 2011).
- While contemporary research regarding procedural knowledge and skills is lacking (Star, 2005), "both procedural knowledge and conceptual knowledge are of critical importance in students' learning of mathematics" (Star, 2007, p. 132).
- Conceptual knowledge is like a web or network of linked relationships of facts. Procedural knowledge means familiarity with the symbols, rules, and procedures for solving mathematics problems (Hiebert & Lefevre, 1986).
- Key to whole number competency is computational fluency—the ability to work efficiently, accurately, and flexibly with numbers (Russell, 2000).



Performance Task in Student *mSpace*

HOW *MATH 180* DELIVERS

The lessons in *Math 180* intertwine conceptual understanding with procedural skill so that students learn the mathematical steps in conjunction with the reasons behind them. Targeted and deliberate practice builds fluency with those procedures. After watching a video that introduces the new concepts in a problem-solving context, the concept is then broken down from concrete to representational to abstract, through the use of interactive mTools and the introduction of mathematical language and symbols. Every lesson in *Math 180* models reasoning and provides rigorous scaffolded practice; as students advance, they build conceptual fluency and model reasoning in order to prepare for high-order performance tasks.

Math 180 offers opportunities for procedural fluency in the Learn Zone and student *mSpace*, where students receive rigorous, scaffolded, and adaptive practice problems. In the Practice (guided practice) section of the software, students receive corrective feedback on each step of every problem, while in the Master (independent practice) section, students receive customized problem sets of gradually increasing difficulty. Students in *Math 180* develop procedural fluency in the Brain Arcade, which provides practice with the procedures, computational skills, models, and strategies in an engaging game environment.

Math 180 provides multiple opportunities to apply the concepts and procedures to real-world situations. Every block contains a career-based, problem-solving mathematics simulation that allows students to apply the concepts they are learning. In the Success Zone, students expand their understanding to nonroutine problems, including word problems, inspired by the next generation assessments. The student *mSpace* provides rigorous, scaffolded practice; students apply their conceptual and procedural understanding on multiple performance tasks of gradually increasing difficulty, all situated in relevant and engaging college and career contexts.

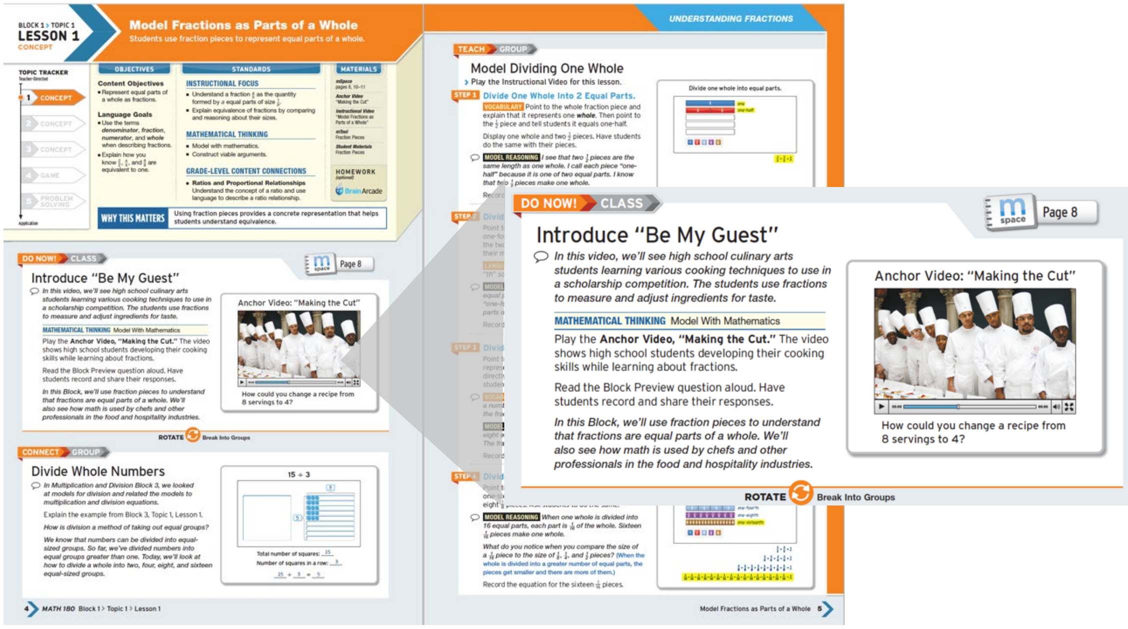
Every Block contains a career-based, problem-solving mathematics simulation which allows students to apply the concepts they are learning.

COMMUNICATION AND STANDARDS FOR MATHEMATICAL THINKING

Any successful mathematics learning environment includes engaged classroom discussions in which every student can clearly communicate their own mathematical reasoning. To foster a communicative classroom culture, *Math 180* provides explicit instruction in mathematical vocabulary and language support, including a Spanish translation of all mathematical terms and audio recordings of all mathematical instruction on the student software. The Standards for Mathematical Thinking are varieties of expertise employed by mathematically proficient students. *Math 180* emphasizes these standards in all aspects of the program.

RESEARCH AND EXPERT OPINION

- It is important for teachers to know and understand the specialized language of math and for students to be provided with challenges that utilize their mathematical language resources (Wilkinson, 2018.
- Communication about mathematical ideas grants an individual and others access to the individual's thinking about mathematical ideas (NCTM, 2000 and supports student learning (Forman, 1996.
- The ability to understand and use language—both conversational and mathematical—is imperative to the development of mathematical concepts (Lager, 2007 and necessary for abstract reasoning in mathematics (Khisty & Morales, 2004.
- The development of mathematical concepts is assisted by students' abilities to understand and use mathematical language (Lager, 2007.
- Learning for understanding involves engaging students with the tools and language of mathematics (Wertsch, 1991) and empowering them to explore mathematics in ways that are meaningful to them (Allsopp et al., 2007; Hiebert et al., 1997).
- Constructive classroom discourse allows students to integrate mathematical tools and language with everyday tools and language (Cobb et al., 1996), focusing on the meaning being developed while the communication occurs (Moschkovich, 2012).
- Teachers should take opportunities to address tensions around language and mathematical content, to establish a mathematical discourse in the classroom community, and to foster the development of academic language and mathematical precision among students (Moschkovich, 2012.
- Teacher modeling of mathematical discourse provides students the opportunity to incorporate mathematical language into their own communications about the mathematics they encounter (Khisty & Chval, 2002).
- To develop students' mathematical proficiency, must make the Standards for Mathematical Thinking an integrated part of student learning (Confrey & Krupa, 2010). The standards support students in order to develop their conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning (NRC, 2001).



Whole-Class Do Now in the *Teaching Guide*

HOW *MATH 180* DELIVERS

Math 180 provides extensive language support and vocabulary instruction designed to allow students to reason abstractly and quantitatively; mathematics vocabulary is both embedded into the instruction at point of use, supporting students' language development, and instantly available to students in a searchable mathematics glossary provided in English and Spanish. Math Talk questions in the Success Zone assess a student's mastery of math vocabulary. Each simulation in the Explore Zone is followed by a reflection exercise, in which students evaluate their thought processes in writing, to be reviewed by classroom teachers in the students' digital portfolio. The student *mSpace* contains an Exit Ticket, which assesses learning through writing.

Math 180 delivers on all the Standards for Mathematical Thinking. At the beginning of every lesson, Do Now activities target and strengthen student skills with specific mathematical practices. Students construct viable arguments by sharing their reasoning, thought processes, and procedures with teachers and with one another in person and in writing.

The Teach section of Group Instruction supports teachers and students as they model problems with mathematics using math games and visual models. Students learn to use appropriate tools strategically both in groups and on the software, where they have full access to the suite of mTools, teach videos, Anchor Videos, metacognitive coaching, and the bilingual math glossary. Finally, students are encouraged to focus on expressing regularity in repeated reasoning in the Learn Zone and the *mSpace* student book, as well as through the use of mTools and in successive levels of games in the Brain Arcade.

Students construct viable arguments by sharing their reasoning, thought processes, and procedures with teachers and with one another in person and in writing.

FORCE MULTIPLIER FOR TEACHING: BUILDING TEACHER EFFECTIVENESS

Guided by Dr. Deborah Ball, the country's most respected voice in effective teaching practices, ***Math 180*** helps teachers become force multipliers by surrounding them with the resources they need to be greater at what they do best. Smart, adaptive technology by Dr. Ted Hasselbring gathers data through every interaction and uses it to personalize learning and provide actionable intelligence to the teacher to make instruction more effective, efficient, and engaging.

Research shows that effective teaching is the driving force behind powerful math instruction and deep understanding. *Math 180* empowers teachers to be force multipliers, dramatically increasing student achievement by providing teachers with the tools, resources, and professional learning they need to improve learning outcomes and create an engaging classroom culture.

Every *Math 180* teacher receives a full suite of professional learning resources: implementation and leadership training and online professional development provided by Math Solutions and High Leverage Practices developed by Dr. Deborah Ball and TeachingWorks. The first two weeks of *Math 180* are designed in conjunction with Mindset Works to develop a mastery-oriented classroom culture in which teachers foster a growth mindset, equating effort with achievement.

Four elements of *Math 180*'s teaching support include

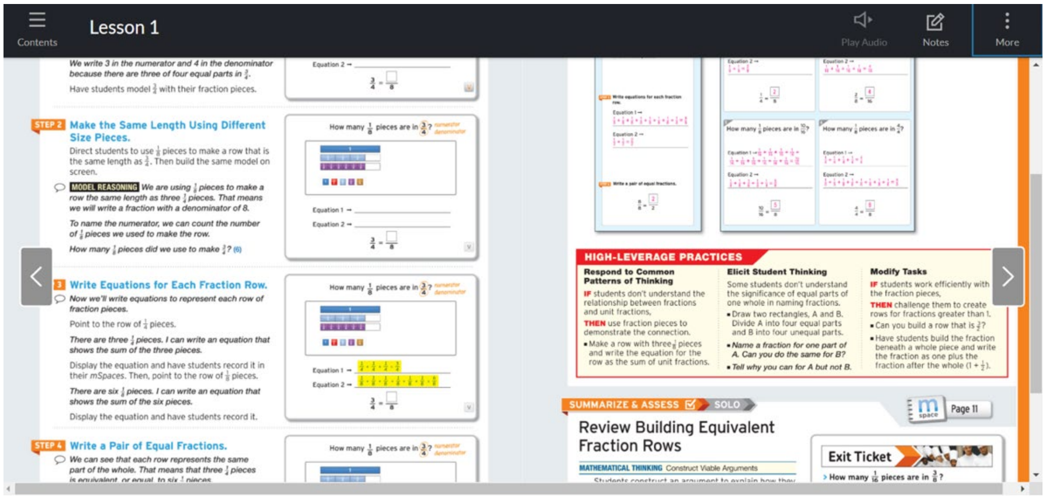
1. **High-Leverage Teaching Practices:** Dr. Deborah Ball outlines best practices to elicit student reasoning, lead discussions, and surface misconceptions.
2. **Data-Powered Differentiation:** Teachers can access actionable data from the student software to create learning groups and differentiate instruction.
3. **Multilingual Learners:** Teachers are provided with language goals and supports to build fluency with academic and mathematical vocabulary.
4. **Students With Special Needs:** In accordance with the Universal Design for Learning principles, teachers receive support for their students with special needs or learning disabilities, including support for individualized education program (IEP) alignment and for active development of the behaviors employed by proficient mathematics learners.

HIGH-LEVERAGE TEACHING PRACTICES

Deborah Ball's High-Leverage Practices for mathematics teaching are designed around the principle that great teachers are not born, but taught. Research-based and proven in thousands of classrooms, the High-Leverage Practices provide a road map for clear, implementable, point-of-use professional learning. In *Math 180*, teachers learn how, why, and when to differentiate at point of use, foster meaningful class discussions, and elicit and respond to student reasoning.

RESEARCH AND EXPERT OPINION

- As classrooms and instruction evolve with new technology, the importance of preparing teachers with the competencies for the digital age must also evolve to meet this challenge (Starkey, 2020).
- "Even teachers who have accurate conceptions of teaching and learning can benefit from a challenge to those conceptions and extend their knowledge" (Ball, 1988).
- A student's educational outlook is determined by the skills possessed by classroom teachers (Ball & Forzani, 2011).
- Teacher change and transforming pedagogy can occur only with proper support and guidance (Hiebert & Grouws, 2007), often necessitating a disruption in their thinking, forcing them to rethink their practices (Cooney, 2001; Zaslavsky, 2005).
- Teachers need sustained professional development opportunities that occur over extended periods of time to reflect on and develop their practice (Grant & Kline, 2004; NRC, 2000; Sowder, 2007).
- Professional development should, over time, develop teachers' mathematical content knowledge, understanding about how students think and learn about mathematics, sense of themselves as teachers of mathematics (Sowder, 2007), and curricular knowledge (Ball, 1988; Remillard & Bryans, 2004).
- Professional development designed for teachers of struggling learners should address beliefs about student mathematical abilities, content knowledge, what it means to "do" mathematics, how students learn, why students struggle, and effective teaching practices. Professional learning should focus on why students struggle and on strategies to address those struggles (Allsopp et al., 2007).
- Teachers should have a deep interest in their students' ideas and thinking about mathematics, as well as their students' content knowledge (TeachingWorks, 2013).
- The National Council of Supervisors of Mathematics recommends that teachers utilize daily warm-up and reflection activities (NCSM, 2013a).



High-Leverage Practices within the *Teaching Guide* online

HOW *MATH 180* DELIVERS

We recognize teachers as the most influential factor in student success. *Math 180* empowers teachers to focus on what they do best—elicit student thinking, develop reasoning skills, and facilitate communication to create an enriching, supportive classroom culture. Dr. Deborah Ball's High-Leverage Practices support teachers as they establish a mathematical mindset, model reasoning, assess student learning across lessons, surface misconceptions, and foster engaging and effective class discussions.

Math 180 lessons highlight exactly which High-Leverage Practices will be the most powerful and relevant at every step of a lesson. One moment may be an ideal time to engage the entire class in a discussion, and another may be a prime opportunity to elicit and interpret an individual student's thinking. Teachers will differentiate at point of use by modifying tasks to match their students' abilities and learn how to establish effective routines for classroom discourse and work. Teachers learn how to respond to common patterns of student thinking with the appropriate strategy or instructional response, correcting misconceptions as they occur.

The High-Leverage Practices are embedded into every *Math 180* lesson, available in the *Teaching Guide* and through HMH's Ed Learning Platform, the teachers' digital dashboard for lesson planning. High-Leverage Practices serve as embedded professional learning, transforming the work of teaching by providing a common vocabulary of effective techniques and strategies across an entire teaching faculty, even those working with multiple curricula and with different grade levels.

"Great teachers are not born. They are taught."

—Dr. Deborah Ball

DATA-POWERED DIFFERENTIATION

***Math 180* is dedicated to providing the resources to educate students with a wide variety of abilities, interests, and learning needs.** Technology-based learning provides assessment and instruction data for every student through ongoing formative assessments and progress monitoring, and *Math 180* translates this data seamlessly into meaningful data snapshots and interactive analytics to target instruction and group students according to their needs.

RESEARCH AND EXPERT OPINION

- Teachers recognize the importance of differentiated instruction and point to its positive influence on motivation, and student-teacher relationships. However, they also point out the challenges in doing it well, indicating the need for proper resources and training (Ginja & Chen, 2020).

- Learning is enhanced when instruction accommodates the differences in learning needs among individual children (Sousa & Tomlinson, 2011).

- Collecting data on student progress to drive instruction is vital to documenting student growth and determining the need for modifying instruction (Stecker et al., 2005).

- Ongoing assessment and progress monitoring are vital to documenting student growth and informing instruction (Fisher & Ivey, 2006; National Joint Committee on Learning Disabilities, 2008; Stecker et al., 2005; Torgesen, 2002).

- Data collected through progress monitoring should provide a clear profile of students' strengths, weaknesses, and needs and should be linked with resources for providing targeted follow-up instruction and intervention (Carnegie Council on Advancing Adolescent Literacy, 2010; National Joint Committee on Learning Disabilities, 2008; Vaughn & Denton, 2008).

- A comprehensive assessment system integrates assessment and instruction, so that educators can continually use data to ensure they are meeting the needs of all students (National Center on Response to Intervention, 2010; Smith, 2010).

- Assessment and instruction data should be used to track student growth, identify students who need more intensive intervention, and assess the efficacy and implementation quality of instructional programs (National Center on Response to Intervention, 2010).

- When students are included in the process of monitoring their own progress, they better understand their academic growth, display increased motivation, and acquire a sense of ownership over their learning (Andrade, 2007, 2008; Forster, 2009; Hupert & Heinze, 2006).

- Findings from a review of the high-quality studies of assessment suggest that the use of formative assessment benefited students at all ability levels. When teachers use the assessment data to provide differentiated instruction, the combined effect is significant (NMAP, 2008).

- Practices central to differentiation, such as grouping students for instruction and engaging learners, have been validated as effective (Ellis & Worthington, 1994).

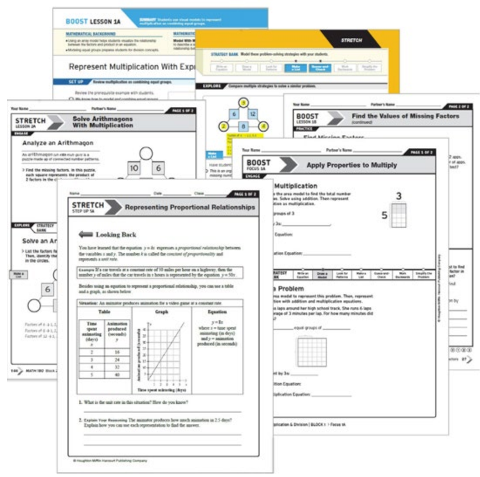
HOW *MATH 180* DELIVERS

In the age of big data, technology is an essential part of the education process. Every moment that a student spends on the *Math 180* software provides continuous, embedded formative assessments and a wealth of actionable data—performance data, formative and summative assessment data, granular data about student timing and engagement, and data about a student's changing mindset and affect toward mathematics.

Teachers receive anytime, anywhere access to student data. From the minute teachers log on to *Math 180* on Ed, their dashboards are populated with Data Snapshots of class performance. Student and classroom analytics allow teachers to see growth and progress toward mastery, and assessment readiness among multiple learning objectives at a glance, all in a digestible fashion designed specificallyto meet the time constraints of working professionals.

In Math 180, analytics are always actionable; teachers are empowered by interactive reports, which recommend the resources and lesson plans that are most effective for each class, group, or individual student.

Recommended lessons are provided both in *Math 180* on Ed and in the *Resources for Differentiated Instruction & Problem Solving Guide* provided in *Math 180*. Targeted boost lesson plans are provided for students who need additional support, while Stretch lessons provide stimulating performance tasks to students who have proven that they are ready for additional challenges.



Student and classroom analytics allow teachers to see growth, progress toward mastery, and assessment readiness among multiple learning objectives at a glance, all in a digestible fashion designed specifically to meet the time constraints of working professionals.

MULTILIGUAL LEARNERS DEVELOPING THE LANGUAGE OF MATH

Students who are learning English may need additional scaffolds to understand mathematical terms. In *Math 180*, instruction begins with progressive language development goals; all mathematical terms are provided in Spanish and English. On the student software many mathematical instructions are translated, and all written instructions are read aloud.

RESEARCH AND EXPERT OPINION

- It is important to recognize multilingual learners are required to learn new content while they are developing proficiency in a new language. The expectations inherent in learning math content points to the important role of language and communication for problem solving (Chval et al., 2021).

- To support the mathematical learning of multilingual learners, instruction should begin with stated content and language goals, build on student cultural context, limit unnecessary language, purposefully develop vocabulary, and utilize cooperative groups as safe learning spaces (Van de Walle et al., 2010).

- Giving multilingual learners a voice in the classroom increases opportunities for teachers to get to know them and assess their readiness to learn. Teachers can thus empower multilingual learners to be successful in the classroom (Avalos, 2006).

- Multilingual learners may be uncomfortable having their struggles with language exposed and may benefit from the private assistance that technology offers (Dukes, 2005).

- Teachers must be cognizant of their lessons' linguistic demands on multilingual learners (Math Solutions, 2011).

- Students learning English as an additional language who are struggling with math must overcome confusion between trying to achieve mathematical understanding and trying to learn mathematical procedures (Frederickson & Cline, 2002).

- Explicit vocabulary instruction is important because students may have existing notions about words like *product, factor, times,* and *sum* that do not align with the mathematical meanings of such words (Allen, 1988; Ball et al., 2005; Garrison & Mora, 1999). The use of manipulatives is especially helpful as multilingual learners master mathematics vocabulary (Garrison & Mora, 1999).

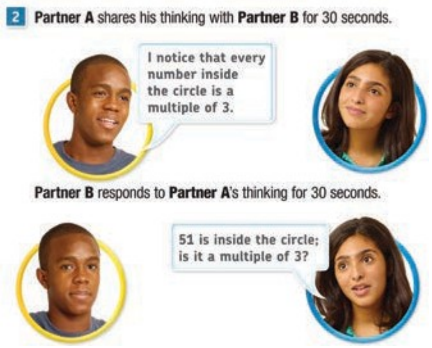
- When multilingual learners are presented with mathematics instruction and techniques that make concepts and operations comprehensible and are presented with additional language and academic support that meets their needs, they are able to be successful. Additionally, working in groups can allow students to develop listening and speaking skills while increasing mathematical understanding (Garrison & Mora, 1999).

- Through classroom discourse, all aspects of mathematical thinking can be discussed, dissected, and understood. Dialogue in the classroom provides access to ideas, relationships among those ideas, strategies, procedures, facts, mathematical history, and more (Chapin et al., 2009).

- The Singapore model of mathematics is taught in English, rather than in a student's mother tongue. This model employs simple vocabulary, which is necessary and effective for helping multilingual learners and struggling students develop problem-solving skills (Ee & Wong, 2002).

HOW *MATH 180* DELIVERS

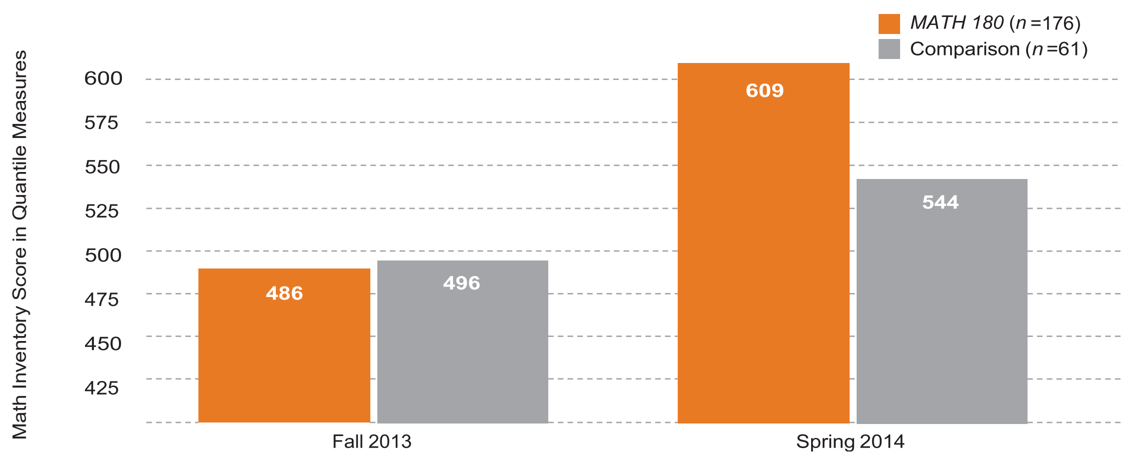
Classrooms with large numbers of multilingual learners or students with low-level language development or literacy face many additional challenges in mathematics instruction. Working with language expert Harold Asturias, the *Math 180* advisors have developed instructional and classroom routines that foster student language learning through mathematical discourse that focuses on consistent academic language.



Math 180 provides progressive language development goals within each section of every lesson, explicitly defined and outlined in all teacher materials. New mathematics vocabulary is introduced during guided practice with a consistent routine of "hear it, see it, say it, and define it." Students discuss, dissect, and understand their mathematical thinking through language-rich classroom routines and discussions. Students write about mathematics and practice expressing their reasoning in the *mSpace*, supported by sentence frames. Language goals culminate in rich performance tasks and multistep word problems.

The student software provides a personal, confidential, risk-free environment in which students can respond, reflect, and access language resources, such as the interactive bilingual mathematics glossary, at their own pace. Every written mathematical instruction in the student software has an accompanying audio recording that can be played aloud multiple times by students with low English-literacy levels. All mathematical concepts are supported by multiple forms of representation: Mathematics tools and vocabulary have visual and oral representations, and engagement is stimulated through images and videos.

COMPARISON OF *MATH 180* SCORES AND SCORE GROWTH FOR STUDENTS DESIGNATED AS ENGLISH LEARNERS



Scholastic. (2014). Early outcome effects of a blended learning model for math intervention with special population students. *Research Update*.

The software support and focus on language goals inherent in *Math 180* have proven to help multilingual learners make significant gains in a study by an independent research team. Multilingual learners using *Math 180* made significantly greater gains on the Math Inventory over their peers in a comparison group.

STUDENTS WITH SPECIAL NEEDS

Students with special needs receive a wide variety of supports in *Math 180* that may be unavailable in traditional classrooms. Instructional tools and methods that have been proven effective for students with learning disabilities— such as immediate corrective feedback at each step of a problem, visual models to promote deep understanding, systematic and explicit instruction of concepts, cooperative and peer-mediated learning techniques, and differentiated lesson design— all contribute to a classroom culture that supports all learners

RESEARCH AND EXPERT OPINION

- Several studies demonstrate that collaborative learning methods such as peer-mediated instruction, produce increased social and achievement benefits for students with and without disabilities (Travers & Carter, 2021).

- A meta-analysis of 50 studies shows that systematic and explicit instruction had a strong positive effect for both special education and low-achieving students (National Council of Teachers of Mathematics, 2007).

- Studies that included visual representations along with other components of explicit instruction produced significant positive effects for students with learning disabilities and low-achieving students (NMAP, 2008). Research demonstrates that dynamic images and sound are especially helpful for students with learning disabilities and other students with limited background knowledge (Hasselbring & Glaser, 2000). “Multiple representations of video information make abstract information more concrete. Video provides students with an authentic base of experience in abstract domains” (Heo, 2007, pp. 31–32).

- Immediate corrective feedback has been found to improve the motivation of mentally delayed adolescents (Distel, 2001; Hall et al., 2000). Successful interventions for secondary students with special needs provide immediate corrective feedback (Vaughn & Roberts, 2007). Immediate, computer-assisted corrective feedback accompanied by answer-until-correct procedures (Epstein et al., 2005) or more practice(Hall et al., 2000) have been found to be effective for students with special needs.

- Universal Design for Learning (UDL) is a set of principles that make learning universally accessible by creating flexible goals, methods, materials, and assessments to accommodate all learners’ differences, including learning disabilities, physical impairments, and sensory impairments. Instructional materials designed with UDL principles increase student access to the curriculum by providing
  - multiple means of content representation to provide students with a variety of ways to learn
  - multiple means of expressing learned content to offer students alternatives to show what they know
  - multiple means of engagement with content to motivate and challenge students appropriately (Rose & Meyer, 2000)

- UDL creates enjoyable and engaging learning environments (Kortering et al., 2008). Teachers applying UDL principles gain a multidimensional view of their students as learners (Howard, 2004).

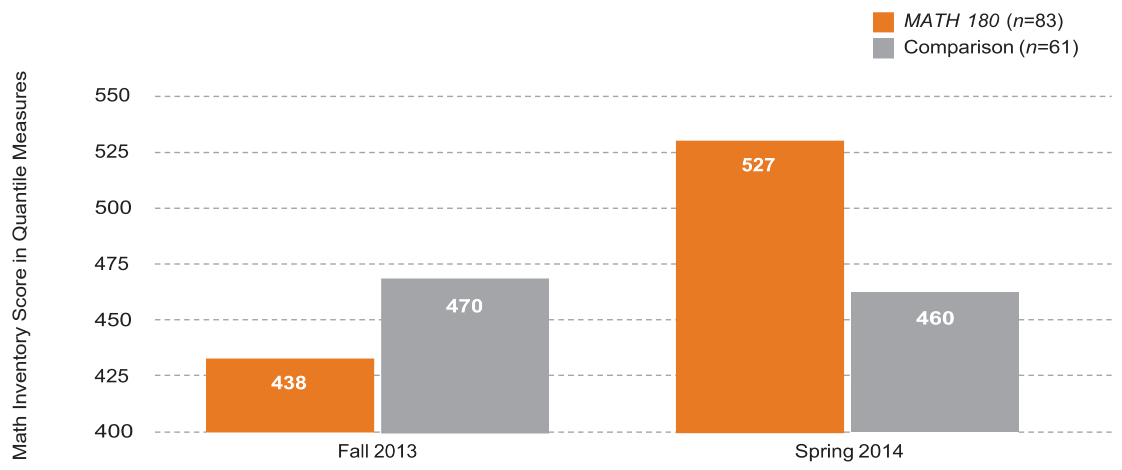
HOW *MATH 180* DELIVERS

Students who have been identified as in need of special education services, or older struggling students, are fully supported in *Math 180.* Individualized data reports allow teachers and parents to continually measure progress against IEP annual goals and benchmarks. The adaptive student software allows students to receive targeted instruction and to move at their own pace, with gradually released scaffolds and a support ecosystem. A study conducted by independent researchers found that after using *Math 180,* students who were designated as having a special education status made significant gains on Math Inventory over their peers who used different mathematics intervention programs.

Working with CAST, UDL principles are integrated into *Math 180* at all levels to facilitate a mathematical growth mindset for all learners. Students consistently encounter math through multiple means of representation, including videos, print, audio recordings, animations, games, and interactive tools on the student software and whiteboard. Students have multiple means of expression and communication with which they can express their mathematical knowledge.

Perhaps most of all, *Math 180* provides multiple means of engagement for learners. Learners understand the value and purpose of mathematics through high-quality videos and animations devoted to real-world career contexts, make choices to express their autonomy in authentic math simulations and Brain Arcade games, and are exposed to a mastery environment through the application of badges, stars, points, and other game mechanics.

COMPARISON OF *MATH 180* SCORES AND SCORE GROWTH FOR STUDENTS WITH SPECIAL EDUCATION STATUS



Scholastic. (2014). Early outcome effects of a blended learning model for math intervention with special population students. *Research Update.*

***Math 180* addresses the unique challenges of students receiving special education services.**

GROWTH MINDSET: ATTITUDES TOWARD INTELLIGENCE CAN IMPACT PERFORMANCE

Math 180 builds student competence and confidence with the help of Dr. Carol Dweck's important research on mindset. In ***Math 180***, teachers are guided to praise students for their effective effort, reinforcing the values of hard work and perseverance. Students learn that making a mistake is a natural part of learning and an opportunity to identify and correct errors.

Students with a fixed mindset about math—a belief that one's ability to understand or excel at math is innate and immutable—do not see the value in putting forth the effort to learn. Struggling students, in particular, are more likely to have a negative, fixed mindset toward their own mathematical abilities (Pashler et al., 2007).

Students with a growth mindset, on the other hand, believe that their knowledge and abilities can be developed and improved over time through effort and dedication. Research states that not only are sustained effort and deliberate practice over time crucial to achieving mastery and expertise but also that students who understand this show more resilience and, ultimately, effectiveness in their pursuit of mastery (Dweck, 2000; Ericsson, 2006; Yeager & Dweck, 2020).

In particular, students who understand that the brain is a muscle that needs to be exercised value practice because they know that such activity will increase both the number and the strength of their neural connections. These same students are more likely to develop a growth mindset and increase their efforts (Blackwell et al., 2007).

Four distinct criteria have been proven to foster a growth mindset:

1. **Purpose and Value:** Students feel that their work is interesting, meaningful, and important.
2. **Agency and Choice:** Students are empowered to make autonomous decisions.
3. **Success and Competence:** Students experience a growing sense of mastery and self-efficacy as a result of their practice.
4. **Community and Family Engagement:** Students receive support for and affirmation of their efforts from others in their immediate environment.

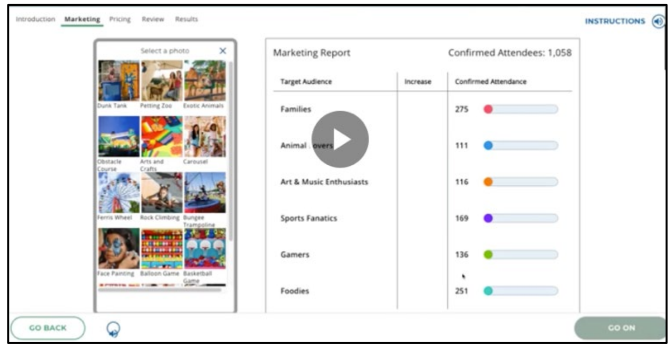
PURPOSE AND VALUE

Although 73% of ACT-tested high school students aspire to some form of postsecondary education such as a two- or four-year college, trade school, or technical school), only 39% of students meet ACT's mathematics benchmark for college readiness (ACT, 2019). When students understand how mathematics is accessible and personally valuable, they become engaged in the work, ready to take responsibility for their own improvement, and increasingly resilient and persistent in their pursuit of mastery.

RESEARCH AND EXPERT OPINION

- Student interest and self-concept in math, which are both important for math achievement, are often at risk for decline as students enter adolescence (Denner et al., 2019).
- When students understand the goals of their work, they are more likely to stay focused, self-monitor, and appreciate their own progress (Rose et al., 2002).
- Students who initially had low expectations for success showed improvement in grades after regularly reflecting on how the material being taught was valuable in their own lives (Hulleman & Harackiewicz, 2009).
- The use of contextually based problems and games provides intrinsic motivation for students (Kamii, 2000).
- Students who believe in the importance of the mathematics they are presented with are more likely to be motivated to do the work necessary to demonstrate understanding. Moreover, the value of the mathematics to a student provides greater motivation than the challenge the mathematics provides unrelated to their goals and aspirations (Schweinle et al., 2006).
- Setting clear goals and expectations increases motivation by encouraging student involvement in and responsibility for their own learning (Ames, 1992; Bransford et al., 2000).
- Work-avoidance behaviors may increase as students perceive their work as lacking meaning (Seifert & O'Keefe, 2001).
- Hrabowski et al. (1998) cite many examples of African American students gaining an interest in their schoolwork when the students can see the purpose behind understanding the lessons.
- Student effort, more than ability, impacts mathematical success (NRC, 2005).

Anchor Video in the student software



Simulation in the student software

HOW *MATH 180* DELIVERS

Math 180 explicitly situates mathematics in college and career contexts, exposing students to a variety of potential visions of future success. Each *Math 180* unit begins with an Anchor Video that presents the upcoming content through high-interest situations. In one Block Series, for example, students learn how the performances of the world's top athletes are often separated by only tenths or even hundredths of a second. In another Block Series Anchor Video, students see how marketers use ratios and social media to highlight the importance of social issues. Whether promoting a social cause or a consumer product, workers in marketing and sales use ratios to convey information to influence thinking and actions.

Every block of *Math 180* contains engaging, media-rich, multistep simulations in which students take on the authentic tasks of social media marketers, restaurant managers, medical and educational professionals, and many other engaging, high-interest careers. Students get to experience the mathematical concepts as concrete, representational, and intensely purposeful before they are asked to perceive them as abstract and algorithmic.

As students put forth effort and progress through the software, they "unlock" a wide variety of potential futures, making an explicit connection between current academic performance and college and career success.

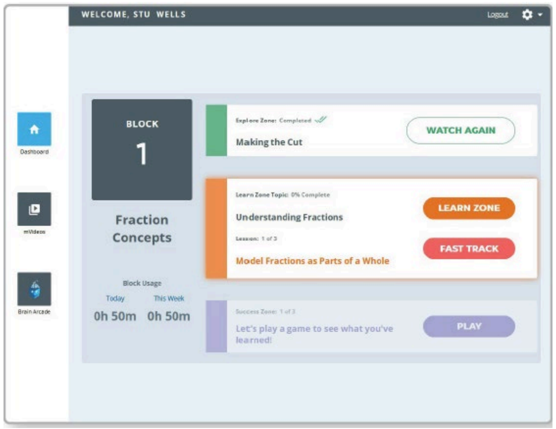
Students get to experience the mathematical concepts as concrete, representational, and intensely purposeful, before they are asked to perceive them as abstract and algorithmic.

AGENCY AND CHOICE

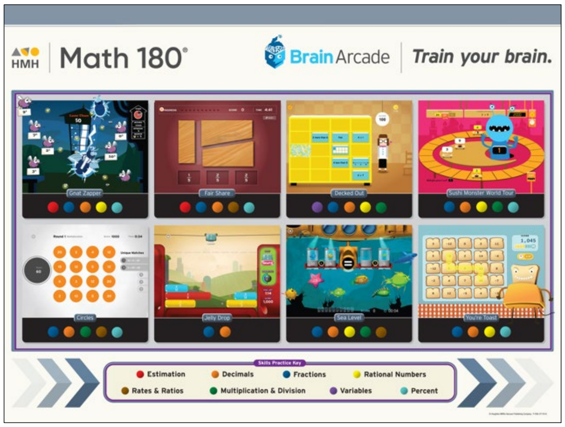
Students who feel a sense of ownership and independence in the learning process demonstrate greater effort, motivation, and engagement with mathematics. However, many struggling students have few chances to make meaningful, autonomous decisions in their daily educational environments. Providing opportunities for agency and personal choice allows students to develop a positive affect and a growth mindset toward mathematics.

RESEARCH AND EXPERT OPINION

- Students tend to have fewer options for choice as they move from elementary to secondary school. However, increasing technology and blended learning environments are opening up possibilities for agency that never existed in traditional classrooms (McCombs, 2015).
- Students who feel they have no control over the outcomes of their efforts are less likely to put forth any effort to learn or improve and demonstrate learned helplessness (Barry, 2007; Murray, 2011).
- Students who attribute failure to uncontrollable factors—such as inability—show little effort or cognitive engagement (Bandura, 1993; Weiner, 1984; Weiner, 1985).
- Self-directed technology, which gives students the opportunity to control the pace of their learning, increases students' sense of independence, motivation, and engagement (Anderson-Inman & Horney, 2007; Hasselbring et al., 2005; Heo, 2007).
- Well-designed educational games, in which students are given agency to explore without risks, can greatly enhance learning. Through autonomous game play, students recognize the value of extended practice and develop qualities such as persistence, creativity, and resilience (Dockterman, 1984; McGonigal, 2011).
- Feeling a sense of autonomy can enhance intrinsic motivation. Students need to feel in control of their choices(Ryan & Deci, 2000).
- Students who had control over their pace and progress performed better on problem-solving transfer tasks than students who could not control pacing (Mayer & Moreno, 2003).
- Students are motivated by choice, control, and challenge (Students at the Center, 2012).



Student Dashboard in the student software



Brain Arcade in the student software

HOW *MATH 180* DELIVERS

Math 180 provides opportunities for students to act autonomously and with agency in every area of instruction. At the onset of each session, students choose which zone of instruction they'd like to work in first. Throughout the Learn Zone, students have the opportunity to attempt an accelerated Fast Track if and when they feel confident and competent to meet the challenge. In the Success Zone, the traditional summative assessment is imbued with agency; students choose which questions and question types to tackle in order to earn points and move forward.

In the Brain Arcade, students can choose among the 11 different math games that improve procedural fluency and strategic thinking. Games are proven to be risk-free environments that destigmatize failure, demarcate progress, and reward persistence, and Brain Arcade games encourage students to pursue their personal mathematical goals with a sense of independence and agency.

Additionally, *Math 180* arms students with multiple strategies for tackling routine, nonroutine, and contextualized problems and then allows students to choose among multiple solution paths. While working through a given problem, students receive immediate feedback that asks them to identify their own mistakes and correct them in real time; this provides them with ownership of the learning process, allowing them to internalize their loci of control and understand that their mistakes are learning opportunities, rather than personal failures. Students are also granted the agency to choose from a variety of scaffolding resources: instructional and Anchor Videos, worked examples, the math glossary, and a library of mTools, virtual manipulatives designed to foster deep concrete and representational understanding.

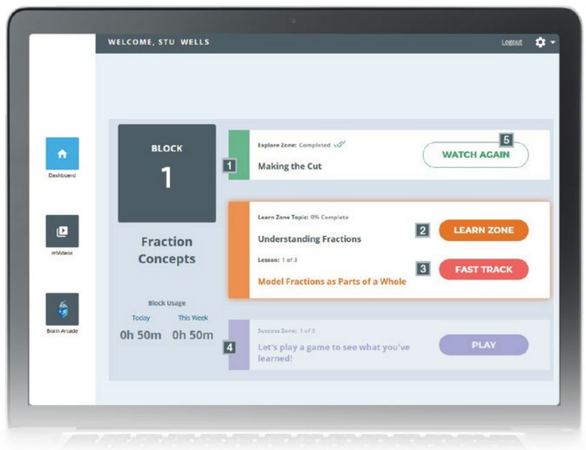
Games are proven to be risk-free environments that destigmatize failure, demarcate progress, and reward persistence.

SUCCESS AND COMPETENCE

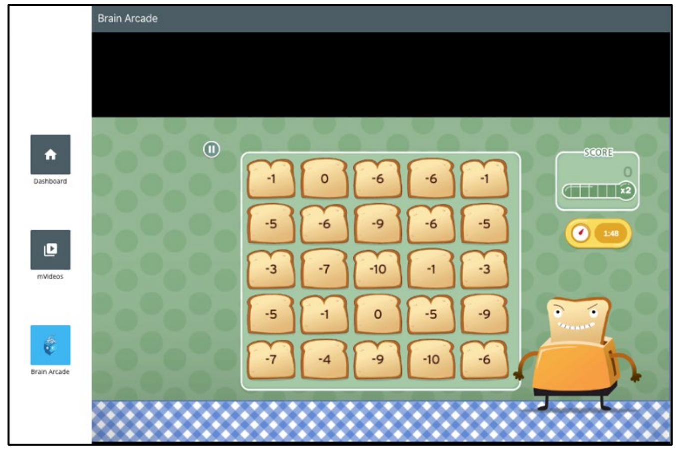
Students who perceive themselves as competent and capable of success are motivated to work persistently toward their learning goals. Conversely, students with a personal history of failure in mathematics, and especially those with a fixed mindset, may perceive themselves as incompetent and unable to improve. ***Math 180*** **provides a safe, supportive environment that rewards improvement; students experience success from the beginning, establishing a foundation of self-efficacy and confidence on which to build.**

RESEARCH AND EXPERT OPINION

- Students who believe their intellectual ability can be developed have demonstrated increased positive outcomes when placed in educational contexts that support this growth (Yeager & Dweck, 2020).
- Students who feel confident will engage in mastery behaviors, such as persisting at difficult tasks, learning from mistakes, and using flexible strategies, while feelings of incompetence can lead students to exhibit performance-oriented behaviors, such as failure avoidance (Dweck, 1986).
- Middle school students who were taught mathematics study skills and that the brain is like a muscle that grows with effort displayed a sharp increase in math achievement in relation to students who were taught only mathematics study skills (Blackwell et al., 2007).
- Students experience greater motivation and confidence when they are aware of their ongoing academic successes. Daily experiences of success greatly increase academic confidence (Pressley et al., 2006).
- “Frequent assessments that start with easier goals and gradually increase in difficulty can build students’ competence and sense of control” (Usher & Kober, 2012, p. 4).
- When students are given ways to feel competent, it becomes more likely that they will learn what is necessary to be successful. In this way, students are able to experience the satisfaction of feeling competent (Sagor, 2003).



Student Dashboard showing block usage



Engaging game design with Brain Arcade

HOW *MATH 180* DELIVERS

In *Math 180,* students experience success early and often, allowing them to establish a baseline of confidence and self-efficacy, even if they have never perceived themselves as competent before. The adaptive software in *Math 180* allows each student to move at their own pace and repeat topics as necessary without any sense of stigma; as the challenge is gradually increased, students are confident in their ability to tackle upcoming topics.

Math 180 builds self-efficacy by tracking and reporting student progress every day and across many dimensions, skills, and attitudes. The Think Tracker shows students that they are progressing with every step of a problem, and the daily Newsfeed, the first thing students see on the software each day, celebrates progress and achievements in every zone of instruction. In the Success Zone, students are assessed not by how many questions they miss, but by how many questions they are willing to take on and answer correctly; each question is rewarded with a variable number of points, tracked by a student-facing progress meter.

Math 180 employs the best practice of game design to convey success, competence, and progress. Students are rewarded with stars and points for their accuracy, focus, perseverance, and effort, as well as for hitting performance goals and showing content expertise. Students receive verbal positive recognition frequently in the program; this encouragement, designed explicitly for students who may not have received recognition for their mathematical efforts in the past, serves as positive reinforcement of mastery that reminds students of their competence and ability to overcome obstacles.

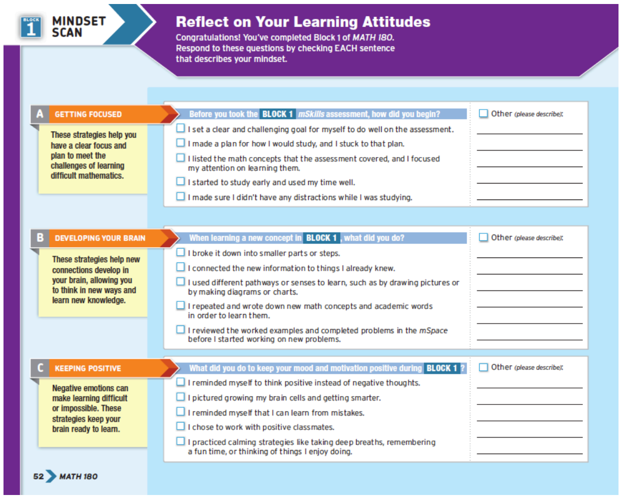
Math 180 employs the best practice of game design to convey success, competence, and progress.

FAMILY AND COMMUNITY ENGAGEMENT

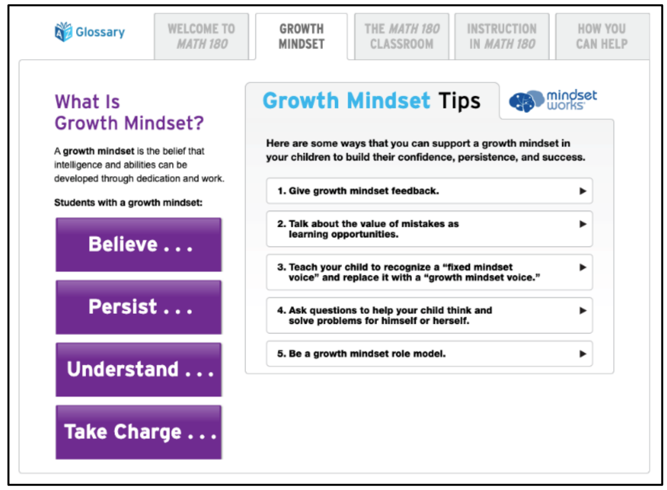
Mathematics instruction does not happen in a vacuum; every student is part of a network of interactions and relationships, and a student's mindset is strongly affected by their peers, teachers, community, and family. *Math 180* provides all the tools to transform the collective mindset of a classroom through class routines, teacher professional learning, and strategies for family engagement.

RESEARCH AND EXPERT OPINION

- Family support has been shown to have a mediating impact on student skills and behavioral engagement, which in turn are predictive of school retention (Gil et al., 2021).
- Students' mindsets are affected by their communities, both inside and outside the classroom. Feedback and classroom discourse can have a lasting impact on how students view intelligence (Burnett & Mandel, 2010). Additionally, perceptions of friends' academic behaviors have a positive correlation with a student's math self-concept and math performance (Jones et al., 2012).
- Partnerships between schools, families, and community are effective for increasing student motivation (Usher & Kober, 2012).
- Teaching is composed of classroom interactions—between the individuals in the classroom and between those individuals and mathematics—that facilitate student learning (Artzt & Armour-Thomas, 1999; Hiebert & Grouws, 2007). These interactions flow from the culture and norms established by the classroom teacher (NRC, 2001) and directed by the learning goals (Stein et al., 2000).
- Programs that engage students in discussions about how people learn, how to overcome obstacles to learning, and how to create a community of learners have shown an increase in students' confidence, motivation, and persistence and a strengthening in students' beliefs that they have control over their intelligence (NCSM, 2010).
- The strengthening of the home-school connection by teachers through sustained communication with families has been correlated with marked improvement in student achievement (Henderson & Mapp, 2002) and growth in cognitive, social, and emotional learning (Cunha & Heckman, 2008).
- Close interaction between parents and teachers, strong parental interest in homework, and a parental view that education is both necessary and valuable are three factors that contribute to the success of African American males (Hrabowski et al., 1998).



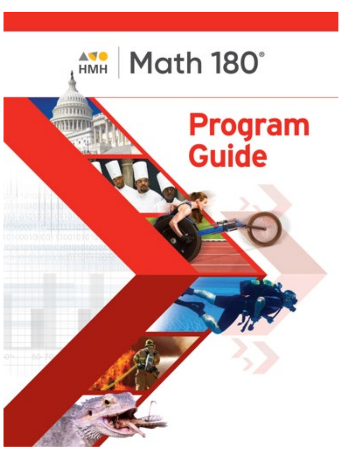
Mindset lessons in the student *mSpace*



Growth Mindset Tips

HOW *MATH 180* DELIVERS

Developed in conjunction with Dr. Carol Dweck's organization, the first two weeks of *Math 180* are dedicated directly to building a classroom culture in which teachers and students deeply understand the principles, language, and tools to foster a growth mindset. Students complete a Mindset Scan at the beginning of the program to establish a baseline for monitoring their own mindsets. *Math 180* then periodically prompts students to reflect on their own mindsets and the learning strategies they are accumulating throughout the program.



Students in *Math 180* work in a safe, supportive classroom environment that values cooperative progress and growth. For teachers, *Math 180* provides lesson plans and classroom routines designed to foster a classroom culture in which students focus on improving their own skills and mastering the material cooperatively. The *Program Guide* and *Teaching Guides* provide teachers with guidance on how to establish vocabulary and principles of growth mindset and transfer them to their students, as well as models of "smart praise" to use with each lesson, in which students are encouraged for their effort, perseverance, and dedication to improvement.

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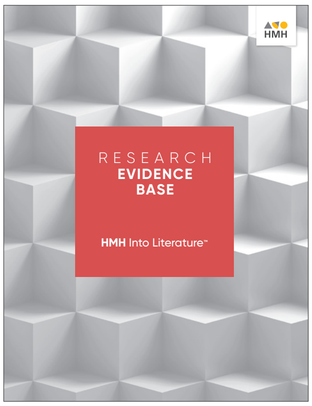

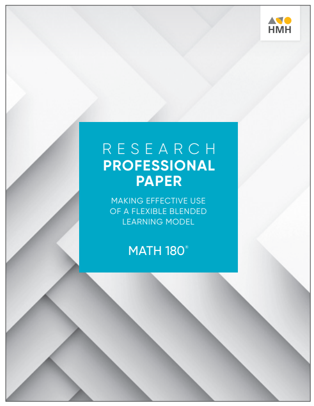

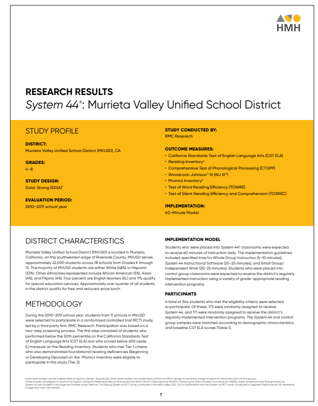

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