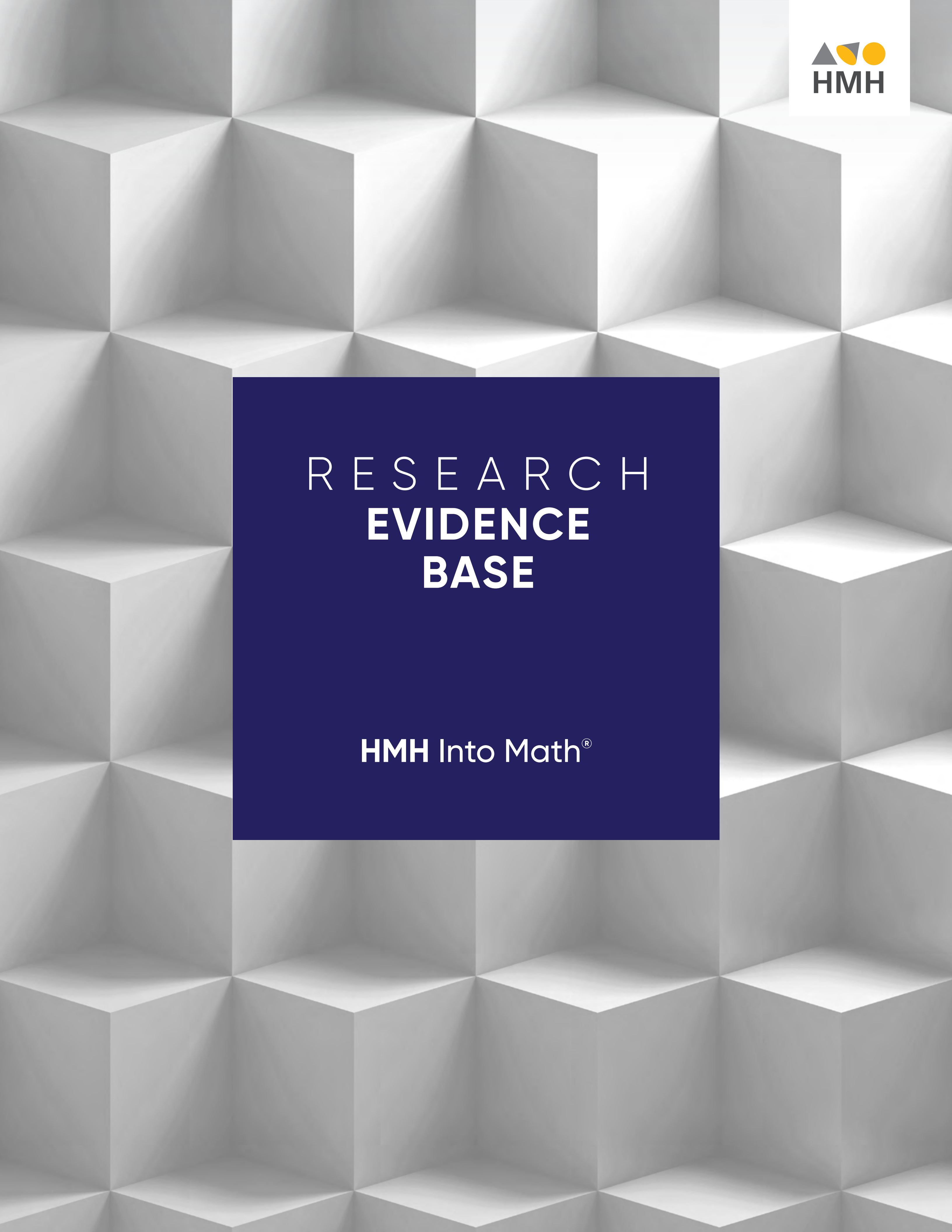


RESEARCH EVIDENCE BASE

HMH Into Math®

THE HMH RESEARCH MISSION STATEMENT

Houghton Mifflin Harcourt® (HMH®) is committed to developing innovative educational solutions and professional services that are grounded in learning science evidence and efficacy. We collaborate with school districts and third-party research organizations to conduct research that provides information to help improve educational outcomes for students, teachers, and leaders at the classroom, school, and district levels. We believe strongly in a mixed-methods approach to our research, an approach that provides meaningful and contextualized information and results.

TABLE OF CONTENTS

INTRODUCTION

Deep understandings of mathematics and well-honed abilities in mathematical thinking are critically relevant for today's students. With careers in STEM increasing significantly over the past decade, it is increasingly important to evaluate the relationship that science, technology, engineering, and mathematics have with each other, especially in terms of math education. Now more than ever, the role of not only technology, but the equitable use of technology, is critical to the success of students in the mathematics classroom and beyond. This level of equity does not only refer to how accessible technology is, but also the premise that, "every student, not just those labeled as honors students, should have the opportunity to engage with high cognitive-demand tasks that used digital mathematical technology" (White, Fernandes, and Civil 2018).

For over a century, the National Council of Teachers of Mathematics (NCTM) has led effort to strengthen math teaching and learning. Their *Principles and Standards for School Mathematics*. describes a vision of equitable and successful school mathematics. It states that this ideal classroom has, "ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment" (NCTM 2000, pg. 3). This focus on equitability, resources, and technology in curriculum is key in makings students are mathematicians in and outside of the classroom.

Houghton Mifflin Harcourt's ***Into Math***TM ***© 2020*** is an intentional, comprehensive, and inspiring mathematics program for Grades K–8 that centers on student growth. Growth is maximized when instruction, assessment, and professional learning are coordinated and tightly aligned. ***Into Math***TM ***© 2020*** is structured to support growth in teaching and learning. The curriculum seeks to promote the following:

- **Focused and Purposeful Content** – Carefully crafted mathematical tasks, differentiated resources, and clear instructional supports help teachers put every student front and center.
- **Ongoing and Relevant Support** – Embedded student supports, classroom videos, resources libraries, and coaching provide learning opportunities for teachers.
- **Integrated and Actionable Assessment, Data, and Reports** – Auto-scored assignments and assessments help educators make data-informed instructional decisions.

Built upon a foundation of mathematics education research and authored by leaders in the field of mathematics education, ***Into Math***TM ***© 2020*** is proven to be effective in raising students' achievement. This document highlights the features of this cohesive, innovative program while explicitly demonstrating the research upon which it is based.

CURRICULUM DESIGN AND STANDARDS

In the modern era, the United States has benefitted greatly from the economic, social, and health advances made possible by a workforce with expertise in Science, Technology, Engineering, and Mathematics (STEM)—and both the importance and demand for jobs in STEM fields continues to increase (Langdon, McKittrick, Beede, Khan, & Doms, 2011). Mathematical conceptual understanding, thinking, and reasoning along with the skills to engage in procedural reliability, fluency, and automaticity are vital capacities for 21st century learners (Granovskiy, 2018). Research demonstrates that standards-based learning environments have a significant positive impact on student achievement in mathematics and that high-performing schools have a clear, focused curriculum in which instruction and assessment are closely aligned to standards (Peterson & Ackerman, 2015; Shannon & Bylsma, 2007; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008). Mathematics programs that effectively support the development of essential 21st century skills are structured by coherent learning progressions that build conceptual understandings as well as connections among areas of mathematical study and between mathematics and the real world (NCTM, 2014).

Dr. Matthew Larson, Past President of the National Council of Teachers of Mathematics, Senior Fellow at Math Solutions, and Author of *HMH Into Math* urges in his post Mathematics Learning: A Journey Not a Sprint (NCTM, 2017) that, while standards initiatives and instructional goals aimed at boosting achievement remain crucial…:

> We must emphasize to parents, teachers, counselors, administrators and students that the goals of learning mathematics are multidimensional and balanced: students must develop a deep conceptual understanding (why), coupled with procedural fluency (how), but in addition they also need the ability to reason and apply mathematics (when), and all while developing a positive mathematics identity and high sense of agency. All four goals are critical components of what it means to be mathematically literate in the 21st century.

HMH Into Math is structured according to coherent learning progressions that utilize evidence-based pedagogy and practices to teach essential mathematics knowledge and skills. Along each grade-level journey, the program fosters within students agency and awareness of their own learning; deep thinking and reasoning abilities; mathematical habits of mind; and language development.

INTENTIONAL DESIGN

To succeed in mathematics, students need a clear, articulated path for learning. [M]athematics instruction—like any good instruction—must be intentionally designed and carefully orchestrated in the classroom, and should always focus on impacting student learning" (Hattie, Fisher, & Frey, 2017, p. 3-4). A coherent math curriculum is sequentially ordered to best reflect the hierarchical and logical structures of mathematics (Schmidt, Wang, & McKnight, 2005). “A robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades” (NCTM, 2014, p.4).

A clear, articulated path toward learning objectives begins with teachers knowing what each student needs to learn each day—and exactly what success looks like for each student (Hattie et al., 2017). Wiggins and McTighe (2005) describe effective instructional design in the classroom as centered on guiding questions, such as: What should students know, understand, and be able to do? How will we know if students have achieved the desired results? How will we support learners as they come to understand important ideas and processes? The authors propose three stages in their model for designing instruction:

- Stage 1: clarifies goals, examines content standards, and reviews curriculum expectations with the purpose of establishing priorities.
- Stage 2: examines the assessment evidence needed to document and validate that the targeted learning has been achieved—a process that further serves to sharpen and focus teaching.
- Stage 3: requires teachers to consider the most appropriate and effective approaches to assessment-based instruction that yields understanding.

Identifying what students will learn is only one aspect of lesson design. It is critical that classroom experiences also connect to what students need to know and makes learning purposeful. Intentional design allows students to recognize, with clarity and intentionality, what is expected of them, including what they are learning and why they are learning it (Kanold, 2018; NCTM, 2014; Wiliam, 2011).

Per Hattie and colleagues (2017), related to intentional design in mathematics is the concept of instructional rigor as viewed as an equally intensive balance among conceptual understanding, procedural skills and fluency, and application. "[M]athematics teaching is most powerful when it starts with appropriately challenging intentions and success criteria" (p. 4).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Today's standards require a focused, coherent, and rigorous curriculum to ensure students develop an in-depth understanding of mathematical concepts and language. Rigorous instruction must include a balanced approach, giving equal emphasis to conceptual understanding, procedural skill and fluency, and application. *HMH Into Math* is a comprehensive mathematics learning system in which all the resources have a clear and intentional purpose that supports effective instruction.

Into Math has three types of lessons, providing teachers a clear focus and a learning objective.

Conceptual These lessons focus on opportunities for students to make sense of the mathematics and build conceptual understanding with real-world context.

- Spark Your Learning
- Build Understanding

Bridging These lessons focus on opportunities for students to connect different conceptual representations, approaches, or strategies to more efficient procedures.

- Spark Your Learning
- Build Understanding
- Step It Out

Procedural These lessons focus on opportunities for students to develop procedural fluency and opportunities for students to apply concepts and procedures.

- Step It Out

Within individual lessons, *HMH Into Math* offers consistent yet adaptable structures and routines that put research-based best practices into action and are augmented by a range of resources to support each student's needs. This intentional design yields dynamic, enriching learning experiences and targeted instructional outcomes.

HMH Into Math's intentional design forms a coherent sequence called a Learning Arc that builds a foundation of conceptual understanding in advance of teaching procedures. These progressions along Learning Arcs also permit connections to students' background knowledge. Opportunities for application are found throughout. An emphasis is placed on connections between concepts and skills. The Learning Arc also ensures delivery of rigorous, relevant instruction.

FOCUSED, PRIORITIZED CONTENT

Reviews of math curricula suggest that a greater focus on fewer core mathematical ideas at each grade yields a greater depth of understanding that results in higher levels of content mastery (Cobb & Jackson, 2011). "The mathematics curriculum in Grades PreK–8 should be streamlined and should emphasize a well-defined set of the most critical topics in the early grades" (National Mathematics Advisory Panel, 2008, p. xiii).

For the past several decades, the cornerstone of education policy in the United States has centered around the implementation of rigorous standards, aligned instruction, and accountability measures. While standards-based teaching with quality materials is demonstrably effective and standards provide a guide to what is critical to teach, standards alone are insufficient in achieving broad improvement to learning. Within standards-aligned instruction, focus and coherence are essential (Schmidt et al., 2005), particularly as they are adapted based on individual students' progress and needs (Pak, Polikoff, Desimone, & Garcia, 2020). Examinations of teaching in American mathematics classrooms concurrent with standards reform efforts have shown a lack of depth and rigor as well as diffuse coverage of content (National Research Council, 2001). In international comparisons of math and science performance, the countries at the top generally present students with fewer topics but at greater depth and increased coherence (National Mathematics Advisory Panel, 2008; Schmidt et al., 2005). "[S]uccessful countries tend to select a few critical topics for each grade and then devote enough time to developing each topic for students to master it. Rather than returning to the same topics the following year, they select new, more advanced topics and develop those in depth" (NRC, 2001, p. 37).

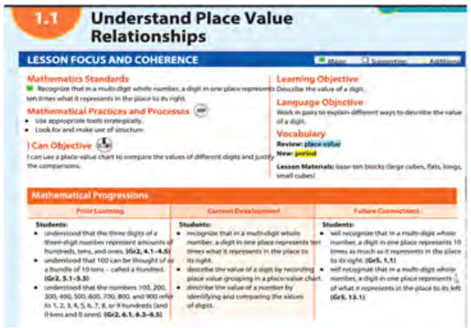
Establishing clear priorities from among national, state, or local content standards is an essential component of instructional planning that will ultimately achieve targeted goals. Standards typically call for more content than can be reasonably, effectively addressed within available time; therefore, teachers must make choices based on the specific needs of their students (Senn, Rutherford, & Marzano, 2014; Wiggins & Tighe, 2005). Additionally, the standards should "promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible" (Achieve, 2010, p. 1).

NCTM (2014) also urges that curriculum design take into consideration the amount of new content to be introduced in a particular grade or course so that sufficient time will be available to teach concepts and procedures using its recommended Mathematics Teaching Practices (which are identified later in this paper). For students to achieve understanding and acquire mathematics skills, identifying and clarifying what those students are expected to learn and understand in a mathematics classroom is an essential component to success (Wiliam, 2011). By addressing the goals within mathematics learning progressions, teachers have the opportunity to examine and monitor student growth in order to adjust instructional priorities as necessary (Sarama, et al., 2004; Sztajn, Confrey, Wilson, & Edgington, 2012).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Mathematics learning and language objectives can be challenging to identify and implement. The *HMH Into Math* solution commits to a concise, logical curriculum, tightly focused on building deep conceptual understanding connected to procedural fluency and thorough application. The solution's Learning Arc supports students in making connections and bridging the conceptual to the procedural, providing them with better access to the concrete models associated with the procedures when they need those procedures to complete more complex tasks.

HMH Into Math offers an articulated curriculum with a clear sequence of content organized by progressions and connected to standards within and across grade levels. The program also outlines essential content and skills and provides teachers with coherent objectives for each lesson of each module.



1.1 Understand Place Value Relationships

LESSON FOCUS AND COHERENCE

■ Major ☐ Supporting ☐ Additional

Mathematics Standards

■ Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

Mathematical Practices and Processes

- Use appropriate tools strategically.
- Look for and make use of structure.

I Can Objective

I can use a place-value chart to compare the values of different digits and justify the comparisons.

Learning Objective

Describe the value of a digit.

Language Objective

Work in pairs to explain different ways to describe the value of a digit.

Vocabulary

Review: place value

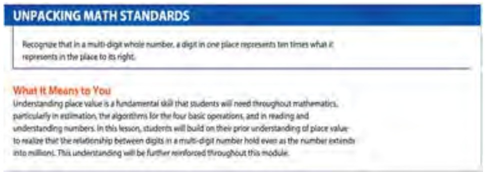
New: period

Lesson Materials: base-ten blocks (large cubes, flats, longs, small cubes)

Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • understood that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. **(Gr2, 4.1–4.5)** • understood that 100 can be thought of as a bundle of 10 tens – called a hundred. **(Gr2, 5.1–5.5)** • understood that the numbers 100, 200, 300, 400, 500, 600, 700, 800, and 900 refer to 1, 2, 3, 4, 5, 6, 7, 8, or 9 hundreds (and 0 tens and 0 ones). **(Gr2, 6.1, 6.3–6.5)**	**Students:** • recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. • describe the value of a digit by recording place value grouping in a place-value chart. • describe the value of a number by identifying and comparing the values of digits.	**Students:** • will recognize that in a multi-digit whole number, a digit in one place represents 10 times as much as it represents in the place to its right. **(Gr5, 1.1)** • will recognize that in a multi-digit whole number, a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left. **(Gr5, 13.1)**

Teacher's Editions include an **Unpacking the Standards** component with interpretative information about standards as well as guidance connecting the standard to other content and objectives. Providing a more thorough description of what is expected and targeted in each lesson allows teachers and students to have a shared understanding of learning.



UNPACKING MATH STANDARDS

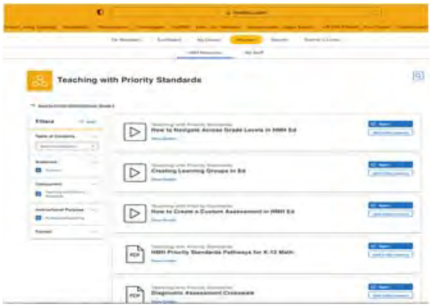
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

What It Means to You

Understanding place value is a fundamental skill that students will need throughout mathematics, particularly in estimation, the algorithms for the four basic operations, and in reading and understanding numbers. In this lesson, students will build on their prior understanding of place value to realize that the relationship between digits in a multi-digit number hold even as the number extends into millions. This understanding will be further reinforced throughout this module.

The Teacher's Editions also include **Language Objectives**. These objectives support students as they learn mathematical concepts and language and practice communicating mathematically.

To further aid the prioritization of content and goals, available to teachers on *Ed*, HMH's online learning platform is a **Teaching with Priority Standards** resource with guidance for educators on a variety of issues and considerations related to this critical process across content areas and specific to math.



Teaching with Priority Standards

How to Navigate Across Grade Levels in HMH Ed

Creating Learning Groups in Ed

How to Create a Custom Assessment in HMH Ed

HMH Priority Standards Pathways for K-12 Math

Diagnostic Assessment Crosswalk

COHERENT LEARNING PROGRESSIONS

Effective mathematics programs feature curricula that develop important mathematical concepts along coherent, meaningful learning progressions and develop connections among areas of mathematical study and between mathematics and the real world. In its expansive research in mathematics teaching and learning, NCTM promotes that the idea that “[m]athematics teachers need to have a clear understanding of the curriculum within and across grade levels—in other words, student learning progressions—to effectively teach a particular grade level or course in the sequence” (NCTM, 2014, p.72).

Learning progressions are a “carefully sequenced set of building blocks that students must master en route to a more distant curricular aim. The building blocks consist of sub skills and bodies of enabling knowledge” (Popham, 2008, p. 83). Because math learning occurs sequentially, building on previous learning and developing in sophistication, part of a discussion of content in mathematics must address the idea of sequence or progressions that promote for students a view of the curriculum as a broader learning process with defined goals for learning. Teachers should support learners as they build on what they know, develop more complex understandings, and realize that mathematics is not a set of discrete parts—it is coherent and connected (Fosnot & Jacob, 2010; Ma, 2010). "[L]earning progressions can be leveraged in mathematics education as a form of curriculum research that advances a linked understanding of students learning over time through careful articulation of a curricular framework and progression, instructional sequence, assessments, and levels of sophistication in student learning" (Fonger, Stephens, Blanton, Isler, Knuth & Gardiner, 2018, abstract).

A coherent math curriculum is sequenced within and across grade levels in a way that best reflects the hierarchical and logical structures of mathematics (Schmidt, et al., 2005).

Beginning in elementary school and continuing throughout their mathematics education, students must develop understanding and use of the "big ideas" that represent overarching concepts as well as specific mathematical reasoning processes essential across domains (Cross et al., 2009, p. 44). The most effective instructional programs will build on children’s intuitive mathematical thinking and use that initial understanding to help children learn to solve problems, employ strategies, and engage in mathematical thinking (Carpenter, Fennema, Franke, Levi, & Empson, 2015). In terms of content, research suggests that for the youngest children, developing a thorough understanding of number and of geometry and spatial measurement are developmentally appropriate and especially crucial to supporting later study (Cross et al., 2009).

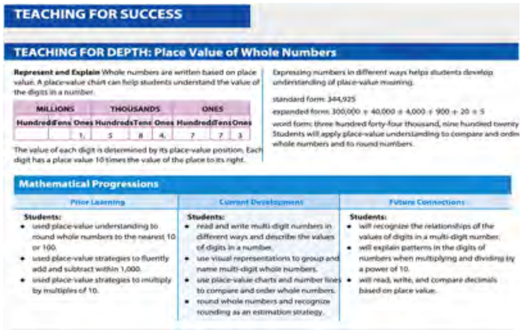
Worth noting, however, is that not everything taught in mathematics fits neatly into a conceptual progression. While there is a temptation “to want to discover universal progressions in learning that are driven by deep changes in conceptual structure . . . there are parts of mathematics learning that, although important and complex, are driven by more incremental mechanisms." This does not suggest, however, that isolated instruction and practice is effective, but rather than there are some mathematical skills which may be best developed with practice in the context of a “meaningful examination of patterns and strategies” (Sherin & Fuson, 2005, p. 385-386).

An essential element in a focused, coherent progression of mathematics learning is an emphasis on proficiency with key topics (National Mathematics Advisory Panel, 2008). To help students build proficiencies, "[d]epending on the learning goals, and where students are in their learning progression, there is a balance of methods that makes for high impact and effective learning" (Hattie et al., 2017, p. 3).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math offers an articulated curriculum with a clear sequence of content organized by progressions and connected to standards within and across grade levels. These progressions reflect the hierarchical and logical structures of mathematics instruction for deep understanding. The program also outlines essential content and skills and provides teachers with coherent objectives for each lesson of each module. Pathways to success are visible to both teachers and students at every step of the way.

HMH Into Math is rigorous, focused, and cohesive, which is necessary for effective mathematics teaching and learning. Throughout the solution, students build their conceptual understandings, improve their procedural fluency, and apply their knowledge in meaningful contexts and real-world applications.



TEACHING FOR SUCCESS

TEACHING FOR DEPTH: Place Value of Whole Numbers

Represent and Explain Whole numbers are written based on place value. A place-value chart can help students understand the value of the digits in a number.

MILLIONS			THOUSANDS			ONES		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
		1,	5	8	4,	7	7	3

The value of each digit is determined by its place-value position. Each digit has a place value 10 times the value of the place to its right.

Expressing numbers in different ways helps students develop understanding of place-value meaning.

standard form: 344,925

expanded form: 300,000 + 40,000 + 4,000 + 900 + 20 + 5

word form: three hundred forty-four thousand, nine hundred twenty

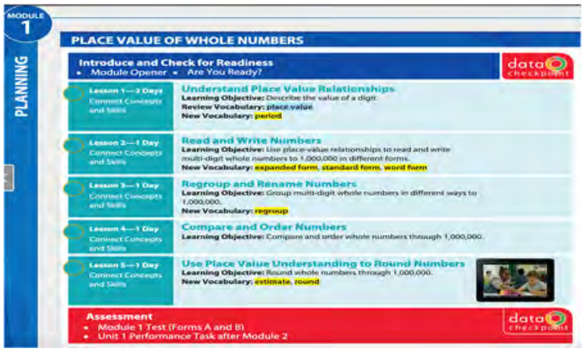
Students will apply place-value understanding to compare and ordi

whole numbers and to round numbers.

Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • used place-value understanding to round whole numbers to the nearest 10 or 100. • used place-value strategies to fluently add and subtract within 1,000. • used place-value strategies to multiply by multiples of 10.	**Students:** • read and write multi-digit numbers in different ways and describe the values of digits in a number. • use visual representations to group and name multi-digit whole numbers. • use place-value charts and number lines to compare and order whole numbers. • round whole numbers and recognize rounding as an estimation strategy.	**Students:** • will recognize the relationships of the values of digits in a multi-digit number. • will explain patterns in the digits of numbers when multiplying and dividing by a power of 10. • will read, write, and compare decimals based on place value.

The **content architecture** is focused, purposeful, and coherent. Each lesson clearly outlines the standards and practices, objectives, and learning progressions.



PLACE VALUE OF WHOLE NUMBERS

Introduce and Check for Readiness
- Module Opener
- Are You Ready?

Lesson 1—2 Days Connect Concepts and Skills	**Understand Place Value Relationships** **Learning Objective:** Describe the value of a digit. **Review Vocabulary:** place value **New Vocabulary:** period
Lesson 2—1 Day Connect Concepts and Skills	**Read and Write Numbers** **Learning Objective:** Use place-value relationships to read and write multi-digit whole numbers to 1,000,000 in different forms. **New Vocabulary:** expanded form, standard form, word form
Lesson 3—1 Day Connect Concepts and Skills	**Regroup and Rename Numbers** **Learning Objective:** Group multi-digit whole numbers in different ways to 1,000,000. **New Vocabulary:** regroup
Lesson 4—1 Day Connect Concepts and Skills	**Compare and Order Numbers** **Learning Objective:** Compare and order whole numbers through 1,000,000.
Lesson 5—1 Day Connect Concepts and Skills	**Use Place Value Understanding to Round Numbers** **Learning Objective:** Round whole numbers through 1,000,000. **New Vocabulary:** estimate, round

Assessment
- Module 1 Test (Forms A and B)
- Unit 1 Performance Task after Module 2

ROUTINES FOR REASONING

Mathematical proficiency requires deep learning. Deep learning requires deep thinking. Deep thinking requires carefully structured, interactive instructional activities that feature predictable and repeatable routines that allow students to focus on and engage with tasks, content, problems, and each other (Lampert, 2015). Research has found that certain instructional routines, many well established and commonly practiced, support the development of mathematical proficiencies, including conceptual understanding, strategic competence, adaptive reasoning, productive disposition, and procedural fluency (Berry, 2018). “Like [classroom] management routines, these ‘mathematical thinking routines’ also have a predictable set of actions that students learn and then practice repeatedly until they are second nature” (Kelemanik, Lucenta, & Creighton, 2016, p.18).

Well-designed routines for reasoning provide essential opportunities for students to articulate complex mathematical situations that allow students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014). "If the goal in mathematics teaching and learning is to support student success with mathematical proficiency, then we must be explicit about using instructional routines that focus on student engagement in activities that support reasoning and sensemaking, communication with and about mathematical ideas, making meaningful connections, building procedural fluency from conceptual understanding, and productive struggle" (Berry, 2018, online). Additionally, effective routines include "low floor/high ceiling" learning tasks, which begin at a level of difficulty that all students can access and attempt and then build in complexity so that ultimately all students are challenged to their individual limits (Sircar & Titus, 2015).

Lampert (2015) recommends that instructional activities regularly include the following elements as part of a predictable routine: providing individual think time for students; having students explain their thinking to one another; having students share their thinking publicly by representing it for the class; and connecting student reasoning to core mathematical thought. "Repeatedly using practices that support these kinds of activities turns important elements of academic engagement into habits. Through repetition, both teacher and students acquire new intellectual and social skills and dispositions. More importantly, perhaps, both teacher and students acquire new ways of thinking about what it means to teach and learn, and what they are able to accomplish" (p. 17).

Lucenta and Kelemanik (2020) further propose an approach to routines for reasoning that centers around mathematical modeling and includes collaborative work through a process of making sense, analyzing the situation, interpreting a model, analyzing and adapting the model, and reflecting on thinking.

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Every lesson of the ***HMH Into Math*** solution represents an intentional design anchored by routines that are consistent yet adaptable to effectively and reliably support students and teachers.

Lessons within each module follow the sequence profiled in the Intentional Design section above. Concepts are introduced and students engage in productive perseverance to explore the concepts. The teacher then assesses student understanding and guides differentiated activities to further develop the concepts for some students and to clarify for others. At lesson's close, students further practice the concepts and procedures, preparing for the next lesson. While ***HMH Into Math*** lessons are organized according to these three stages, they refine teaching and learning to accommodate specific needs as they arise and as assessment deems. Specific components of the routine accomplish the following aims:

Spark Your Learning: Teachers work on-level with students to gauge their readiness and to inspire and guide productive perseverance. In Apply and Practice lessons, Spark Your Learning is replaced with Step It Out to help students begin to build fluency, learn to choose from multiple available strategies, and rely on the conceptual understanding developed previously to solve rigorous tasks. These tasks also provide accessible **low floor/high ceiling mathematics instruction** to meet individual students at their optimal learning level.

Learn Together: Whole-group learning is facilitated in these Build Understanding and Step It Out tasks. Build Understanding tasks provide an opportunity to help students understand lesson concepts. Step It Out tasks promote procedural understanding.

Check Understanding: After the learning tasks, these five- to ten-minute checkpoints provide a snapshot of what students know.

Differentiation Options: According to student understanding, groups are formed to ensure growth for each and every student by providing resources based on individual needs. Teachers can then decide how to best support students with differentiated resources such as independent practice, Math Centers, or connecting to the Teacher Tabletop Flipchart mini-lesson and additional small-group activities.

Wrap-Up: Here is the opportunity for additional practice, reteaching, or intervention. Teachers gauge student depth of understanding with exit tickets and suggested wrap-up ideas.

Homework or Practice: Each lesson includes homework/practice opportunities for students to practice the concepts just introduced.

Mathematical language routines are also provided throughout ***HMH Into Math***. These language routines serve to amplify, assess, and develop students' language skills and usage through ongoing, predictable, flexible opportunities for students at all language proficiency levels to listen, speak, and write about mathematical situations. The ***HMH Into Math*** mathematical language routines feature is profiled in the Embedded Language Development and Support section that follows.

Every ***HMH Into Math*** lesson provides ample opportunities for teachers to engage students and check students' understanding as it develops. Every lesson allows for students to practice what they are learning, refine their problem-solving skills, and showcase their growing positive mathematical mindset and skill set. The solution is intentionally designed to reflect the realities of actual classrooms and support individual student needs—while achieving the rigorous, standards- and research-based goals for learning.

MATHEMATICAL HABITS OF MIND

"The ability to solve new and unforeseen problems requires mastery not just of the results of mathematical thinking (the familiar facts and procedures) but of the ways that mathematically proficient individuals do that thinking. This is especially true as our economy increasingly depends on fields that require mathematics. Mathematical proficiency depends also on other mental habits that dispose one to characterize problems (and solutions) in precise ways, to subdivide and explore problems by posing new and related problems, and to 'play' (either concretely or with thought experiments) to gain experience and insights from which some regularity or structure might be derived" (Goldenberg, Mark, Kang, Fries, Carter & Cordner, 2015, p. 1-2).

Researchers have advocated for using mathematical habits of mind as a framework for approaching math instruction for several decades but the idea is timeless: mathematics has always been about more than its products—facts, methods, formulas, etc.—as successful study within the field draws on cognitive practices such as strategies and behavioral dispositions such as perseverance to solve complex problems (Cuoco, Goldenberg, & Mark, 1996; Goldenberg et al., 2015). In their seminal work, Cuoco and colleagues (1996) proposed that "more important than specific mathematical results are the habits of mind used by the people who create those results . . . this includes learning to recognize when problems or statements that purport to be mathematical are, in truth, still quite ill-posed or fuzzy; becoming comfortable with and skilled at bringing mathematical meaning to problems and statements through definition, systematization, abstraction, or logical connection making; and seeking and developing new ways of describing situations" (p. 376).

Mathematical habits of mind reflect how mathematicians think about situations in automated, internalized ways that allow them to persist through complex problems. Developing mathematical habits of mind is essential to mathematical proficiency, critical thought, college and career readiness, access to future opportunities, and productive participation in society (Goldenberg et al., 2015). "If we really want to empower our students for life after school, we need to prepare them to be able to use, understand, control, modify, and make decisions about a class of technology that does not yet exist. That means we have to help them develop genuinely mathematical ways of thinking" (Cross, Woods, & Schweingruber, 2009, p. 21).

Mathematical habits of mind develop as a by-product of teaching mathematics through problem solving, in a process that entails modeling and reflection so that habits are internalized (Kaplinsky, 2018 & 2019; Levasseur & Cuoco, 2009). Effectively problematizing mathematics has students think for themselves and explain their thinking while also supported by their teacher, classmates, and math program; to struggle productively; and ultimately to apply their gained knowledge and strategies to new and more complex problems they encounter in the future (Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, & Wearne, 1996). Ultimately, problem solving in the mathematics classroom encourages students to see that their actions can lead to intellectual growth, and this "focus on the potential of students to develop their intellectual capacity provides a host of motivational benefits" (Blackwell, Trzesniewski, & Dweck, 2007, p. 260).

To cultivate mathematical habits of mind, teachers also must create a classroom culture that demonstrates how challenge is a natural part of the learning process (Star, 2015) and allows students to see the benefits of perseverance (Hiebert & Grouws, 2007). Educators and students both must also adopt growth mindsets and positive views on productive challenge. These attitudinal states yield numerous desired affective outcomes and boost academic achievement (Dweck, 2006, 2008 & 2015; Hiebert & Grouws, 2007; NCTM, 2014).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Throughout the program, *HMH Into Math* cultivates students' mathematical habits of mind. It does so via a multipronged approach that emphasizes problem solving in real world applications, making the math learning relevant within and beyond the classroom. *HMH Into Math* also fosters productive perseverance, helping students to persist through process and see the process itself as essential and enjoyable. The program also helps students see themselves as capable problem solvers and math learners through such features as the **I Can** statements that introduce problem solving tasks.

I can compare and order three numbers to the hundred thousands place. I can record and justify the comparisons.

The Library of Congress, in Washington, D.C., is the largest library in the world. It has about 700,000 rare books, about 124,000 telephone books, and about 140,000 comic books. How can you compare the numbers to find which kind of book there is the most of at the Library of Congress?

The program also develops students' productive perseverance with the aid of strategies from **Mindset Works**.

I'm in a Learning Mindset!

What did I learn from peers when they shared their strategies with me for writing a unit rate?

HMH Into Math further fosters mathematical habits of mind by extending the learning journey. As part of differentiation tools, the program provides a **Ready for More** small group option that allows students to expand their knowledge and additionally explore topics and their real-world implications, impact.

Ready for More

Materials: internet, newspapers, magazines

Ask students to think about real-world situations where people might want to compare numbers. Then invite them to research these situations and make a poster to illustrate the comparisons. For example, have students compare appliance prices in ads, sports scores, or grocery prices, as well as greater numbers such as local and state populations, costs to travel internationally, and so on.

EMBEDDED LANGUAGE DEVELOPMENT AND SUPPORT

Multilingual learners are the fastest growing student population in the United States, representing about 4.5 million or nearly 10% of overall enrollment in public schools during the 2013-14 academic year (Grapin, 2019; National Center for Education Statistics, 2016). The description "multilingual learner" applies to all students who regularly interact with languages other than English, including but not limited to those commonly referred to as English language learners (ELs). "Multilingual learners come from a wide range of cultural, linguistic, educational, and socioeconomic backgrounds and have many physical, social, emotional, experiential, and/or cognitive differences. All bring assets, potential, and resources to schools that educators must leverage to increase equity in standards-based systems. Increasing avenues of access, agency, and equity for all multilingual learners—including newcomers, students with interrupted formal schooling (SIFE), long-term English learners (L-TELs), students with disabilities, and gifted and talented English learners—requires educators to be knowledgeable, skillful, imaginative, and compassionate" (WIDA, 2020, p. 18).

In a 2020 practice brief based on current research findings, U.S. Department of Education's Office of English Language Acquisition identifies five key practices for educators teaching English learners:

1. Embrace asset beliefs that position and support ELs as full participants in mathematical learning.
2. Engage ELs in meaningful interactions and discourse with others.
3. Provide support for ELs to engage in mathematical practices.
4. Sustain an explicit focus on language as it is used in math.
5. Design mathematical learning experiences that engage ELs in rich communications integrating oral and written language.

"Students who are not fluent in English can learn the language of mathematics at grade level or beyond at the same time that they are learning English when appropriate instructional strategies are used...Effective mathematics instruction leverages students' culture, conditions, and language to support and enhance mathematics learning (NCTM, 2014, p. 63)"

While many perceive that math is easier for ELs to learn because it involves numbers, mathematics actually presents specific language challenges to this student population (Janzen, 2008). It is important for all students, but particularly multilingual students, to regularly communicate verbally and in writing about their mathematical ideas, and in doing so that "they not only reflect on and clarify those ideas but also begin to become a community of learners" (Bray, Dixon, & Martinez, 2006, p. 138). Also, when introducing academic words to English language learners' expressive vocabularies, students respond best to classrooms that offer predictable routines and frequent, comfortable opportunities to express what they have learned (Feldman & Kinsella, 2008). Connecting math and language through productive struggle is also important for ELs (Asturias Méndez, 2015).

Zwiers, Dieckmann, Rutherford-Quach, Daro, Skarin, Weiss, and Malamut (2017) developed a framework for promoting language and content development in tandem within mathematics instruction. The framework is based on four design principles to guide curriculum planning and teaching practices:
1. Support sense-making: Scaffold tasks and amplify language so students can make their own meaning
2. Optimize output: Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing.
3. Cultivate conversation: Strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class).
4. Maximize linguistic and cognitive meta-awareness: Strengthen the "meta-" connections and distinctions between mathematical ideas, reasoning, and language.

Additionally, Zwiers and colleagues' framework recommends eight research-based mathematical language routines (MLRs) with structured but flexible formats that emphasize meaningful and purposeful English and domain-specific language use:

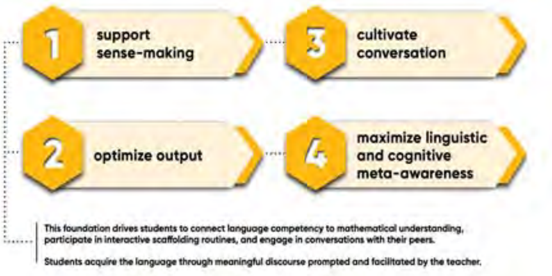
- MLR1: Stronger and Clearer Each Time: to provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output.
- MLR2: Collect and Display: to capture students' oral words and phrases into a stable, collective reference containing illustrations connected to mathematical concepts and terms.
- MLR3: Critique, Correct, and Clarify: to give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.
- MLR4: Information Gap: to create a need for students to communicate in math.

- MLR5: Co-Craft Questions and Problems: to allow students to get inside a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.
- MLR6: Three Reads: to ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning.
- MLR7: Compare and Connect: to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.
- MLR8: Discussion Supports: to support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies.

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

A driving mission within *HMH Into Math* is to present all students, including and especially English learners, with frequent opportunities to speak, write, read, and listen in the mathematics classroom to boost the development of their language skills, both specific to the domain and generally. The program provides explicit, deliberate, evidence-based, and WIDA-aligned support within a mathematics context, utilizing strategies to produce growth for students within each level of language proficiency.

HMH Into Math uses current research to challenge traditional language development practices in the classroom and highlight the importance of intentional materials, design, and professional learning. *HMH Into Math* was designed with four guiding principles to support mathematical language use and development. The four design principles, infused within Mathematics Language Routines, are based on the work of Zwiers and colleagues (2017) cited above and they serve as the foundation for the language development practices in *HMH Into Math:*



This foundation drives students to connect language competency to mathematical understanding, participate in interactive scaffolding routines, and engage in conversations with their peers.

Students acquire the language through meaningful discourse prompted and facilitated by the teacher.

HMH Into Math's **Routines for Language Development** help teachers promote the design principles routines that are structured, but adaptable, in a format for amplifying, assessing, and developing students' language skills and usage. These Routines provide opportunities for students to listen, speak, and write about mathematical situations with practices that are appropriate and effective for all language proficiency levels. Routines include these features:

- **Three Reads** –To ensure understanding of mathematical questions, students read a problem three times with a specific focus each time addressing Mathematical Practice 1 (MP.1).
- **Stronger and Clearer Each Time** – Students use structure to write their reasoning behind a problem, share and explain their reasoning, listen to and respond to feedback, and then write again to refine their reasoning (MP.6).
- **Compare and Connect** – Students listen to a partner's solution strategy and then identify, compare, and contrast this strategy application, in a process that boosts metacognitive awareness (MP.6).
- **Collect and Display** – Students capture oral words and phrases learned and build a collective reference containing illustrations connected to mathematical concepts and terms within each module (MP.6).
- **Critique, Correct, and Clarify** – Students correct work that is not their own with a flawed explanation, argument, or solution method and share with a partner to reflect and then refine the sample work (MP.3).
- **Teacher Tabletop Flipcharts** contain leveled scaffolding and support for English Learners. These tools ensure teachers maintain the rigor and cognitive complexity level required for mathematical reasoning while supporting ELs.

HMH Into Math allows the acquisition of academic vocabulary to emerge after the students explore a concept and develop understanding. Rather than front-loading new vocabulary, it is highlighted after the concept is taught, connecting students' understanding of the concept to the explicit vocabulary term. With this unique approach, English learners simultaneously boost their disciplinary and English language abilities.

HMH Into Math **Readers** allow teachers to integrate literature into their math instruction. An assortment of titles is provided for each grade-level. *HMH Into Math* **Readers** help students build mental models for abstract concepts and strengthen students' reasoning and conceptual understanding. They also enhance academic vocabulary and bring engaging mathematical content to life. Into Math Readers are available in print and online. A copy of each grade-level title is provided in the program's differentiated resources kit. Digital e-books with audio support are included within HMH's *Ed* platform. Spanish versions of the Readers are available as well.

Linguistic Notes are provided in the **HMH** ***Into Math*** Teacher's Edition to support teachers with cues for what to listen for, tips to prevent language misunderstanding, and repeated opportunities to elicit students' their mathematical thinking. The notes help teachers with ideas in how to best support English learners in the classroom and improve language development alongside mathematical content.

MATHEMATICAL KNOWLEDGE AND TEACHING

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 7). Effective teaching and its development of students' mathematical knowledge are the driving forces behind powerful mathematics instruction and deep understanding. Research continually demonstrates that mathematics learning should be focused on engaging students in instructional tasks and interactive practices that promote reasoning, problem solving, and discourse—all with the aim of fostering understanding of mathematical concepts and procedures (NCTM, 2009 & 2014; National Research Council 2012).

HMH Into Math empowers teachers by providing them with the tools, resources, and professional learning they need to improve outcomes and create an engaging classroom culture. *HMH Into Math* aligns with the National Council of Teachers of Mathematics (NTCM)'s (in *Principles to Actions: Ensuring Mathematical Success for All*, 2014) framework of eight essential, research-based, high-leverage practices for teaching and learning that promote deep understanding of mathematics, as described in the section that follows.

CLEAR GOALS TO FOCUS LEARNING

Research demonstrates that clarity between teachers and students regarding intentions for what is to be learned, why it is to be learned, and criteria for what constitutes success is one of the most effective teaching practices for yielding targeted outcomes (Almarode & Vandas, 2018; Hattie, 2009; Leahy, Lyon, Thompson, & Wiliam, 2005). "Formulating clear, explicit learning goals sets the stage for everything else" (Hiebert, Morris, Berk & Jansen, 2007, p. 57). Setting specific goals and expectations that articulate a clear path for behavior and desired performance serve for students as motivation for learning and a sense of greater agency (Bransford, Brown, & Cocking, 2000; Marzano, 2010). Additionally, promoting self-determination is an important component in classroom instruction aimed at helping all students attain post-academic success and quality of life, and particularly for students with special needs, helping students develop skills associated with self-determination—e.g., planning, self-management, self-awareness, problem-solving, and goal-setting—is critical in preparation for experiences within and beyond school (Raley, Schogren, & McDonald, 2018).

"Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions" (NCTM, 2014, p. 3). Pointing out that, historically, "piecemeal efforts aimed at narrow learning goals have failed to improve U.S. students' learning" (p. 12), NCTM calls for mathematics teaching that develops understanding through coherent curricula that sequence core mathematical ideas into learning progressions. "[A] well-articulated curriculum gives teachers guidance regarding important ideas or major themes, which receive special attention at different points in time," as, specifically, "…it must be coherent, focused on important mathematics, and well-articulated across the grades" (NCTM, 2014, p. 14). A coherent math curriculum is sequentially ordered to best reflect the hierarchical and logical structures of mathematics (Schmidt, Wang, & McKnight, 2005). Because math learning occurs sequentially, builds on previous learning, and develops in sophistication, mathematics education must address the idea of progressions that helps students see a curriculum as a broader learning process with defined goals for learning (Marzano, 2009). Learning progressions are carefully sequenced sets of sub-skills and bodies of enabling knowledge that students must master to reach more distant curricular goals (Popham, 2007).

By looking at the goals within mathematics learning progressions, teachers have the opportunity to examine and monitor student progress and needs in order to adjust instruction as necessary (Charles, 2005; Sarama, DiBiase, Clements, & Spitler, 2004; Sztajn, Confrey, Wilson, & Edgington, 2012). Teachers can support learners as they build on what they know, develop more complex understandings, and realize that mathematics is not a set of discrete parts; rather, it is coherent and connected (Fosnot & Jacob, 2010; Ma, 2010). For students to achieve understanding and acquire mathematics skills, identifying and clarifying what students are expected to learn and understand in a mathematics classroom is an essential component to success (Wiliam, 2011)—and a failure to provide clear expectations results in low levels of achievement (Black & Wiliam, 1998a).

Work by Haystead & Marzano (2009) and Hattie (2009) shows that students in classrooms where learning goals are clearly articulated perform at higher levels than students who are unaware of the expectations. While it is important for learning goals to be clear, it is equally important that students are the ones doing the "sense-making" (Dixon, 2018). Indeed, establishing clarity for learning aims is ideally an authentically co-creative process between teachers and students (Almarode & Vandas, 2018). When expectations are discussed with their teachers, students are able to find value in their work and understand the greater purpose of what they are learning (Black & Wiliam, 1998a; Marzano, 2010).

Additionally, establishing goals allows students to focus on set expectations and become more aware of their own thinking and learning (Clarke, Timperley, & Hattie, 2004). Curriculum designed and developed for 21st-century learning makes learning goals transparent to students; continuously monitors, provides feedback, and responds to students' learning progress toward goals; and engages students in self- and peer assessment in achieving goals (Committee on Defining Deeper Learning and 21st Century Skills, 2012).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Within *HMH Into Math*, the what, how, and why associated with learning tasks is transparent and relevant to students. Expectations are applied and reinforced throughout lessons, including in discourse with peers through Turn and Talk.

Module

2

Addition and Subtraction of Whole Numbers

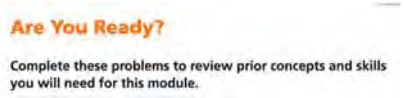
How can you reason in reverse?

- A filmmaker wants to record scenes on the roofs of three buildings. She uses a drone with a camera.
- Suppose the drone starts on the roof of one building, flies down to the roof of a second building, and then flies up to land on a third building, as shown.
- How tall is the first building? ____________

Turn and Talk

- How does the relationship between addition and subtraction help you solve this problem?

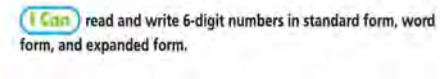
Expectations around assessment activities are also clarified for students.



Are You Ready?

Complete these problems to review prior concepts and skills you will need for this module.

The I Can feature in the *HMH Into Math* Student Edition at each grade identifies specific expectations for learning as well as reinforces a sense of competence and capability. <<asset from Into Mat Gr4 SE p11>>



I Can read and write 6-digit numbers in standard form, word form, and expanded form.

REASONING AND PROBLEM SOLVING

"[S]olving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which is not immediately attainable" (Polya, 1965, p. ix). Problem-solving is an engrained and essential process within human experience. In the discipline of mathematics specifically, activities such as formulating problems and assessing the reasonableness of various approaches to their solutions is central to the development of skills and knowledge (Santos-Trigo, 2020). Engaging students in problem-solving tasks allows them to actively construct mathematical understandings and more deeply and with greater meaning than when teachers present information to students and have them carry out procedural exercises (Masingila, Olanoff, & Kimani, 2018). "Even the clearest teacher explanations leave many students with incomplete understanding and shaky confidence. Ideas that are forged by hard thought and tested in discourse with other students and teachers are much more likely to last and be useful" (Marcus & Fey, 2003, p. 61).

To cultivate critical thinking capacities and develop mathematical concepts, students need regular opportunities to be challenged by problem solving tasks with multiple paths to the solution (Kaplinsky, 2019). High-quality, research-based instructional math programs build on students' intuitive mathematical thinking and unique background knowledge; incorporate rich and rigorous problem-solving tasks that engage interest; require that students employ strategic thinking and mathematical habits of mind—all with the larger aim of developing, over time, students' conceptual understanding and procedural fluency (Carpenter, Fennema, et al., 2015; David & Greene, 2007; Hiebert et al., 1996; NCTM, 2014). "Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014, p. 17).

Task selection is a critical aspect of supporting elementary students' reasoning and understanding in mathematics—and among the key features of effective instructional tasks are that they be challenging and connective and as well as open to multiple representations and multiple strategies for solutions (Childs & Glenn-White, 2018; Francisco & Maher, 2005; Maher, 2002; Mueller, Yankelewitz, & Maher, 2014). Tasks that consistently encourage high-level student thinking and reasoning (versus those that are routinely procedural) yield the greatest learning; and tasks of higher cognitive demand are necessary when promoting reasoning and problem solving in the mathematics classroom (Miri, David, & Uri, 2007; NCTM, 2014; Stein & Lane, 1996). In constructing mathematical tasks, it is further recommended that teachers "problematize with the goal of understanding the situations and developing solution methods that make sense" (Hiebert et al., 1996, p. 19).

Students learn best when what they learn is relevant and meaningful. Connecting problem solving tasks to real world contexts and applications improves perceptions of the content as interesting and beneficial, thereby increasing motivation to learn (Czerniak, Weber, Sandmann, & Ahem, 1999). "When instruction is anchored in the context of each learner's world, students are more likely to take ownership for . . . their own learning" (McREL, 2010, p. 7). Students at all levels need to connect the mathematics they are learning to the world around them (Alberti, 2013) and teaching with contextual problems can be effective for developing "children's mathematical modeling of the real world" (Fosnot & Dolk, 2010, p. 24).

Making connections between new information and students' existing knowledge— knowledge of other content areas and of the real world—has proved to be more effective than learning facts in isolation (Beane, 1997; Bransford et al., 1999; Caine & Caine, 1991; Kovalik & Olsen, 1994). Further, connecting mathematics to science, social studies, and business topics can increase students' understanding of and ability with mathematics (Russo, Hecht, Burghardt, & Saxman, 2011). In their study of mathematics learning in early childhood, Cross, Woods, and Schweingruber (2009) concluded that to effectively foster students' conceptual understanding, teachers must include four key elements or opportunities within their teaching and learning activities: analyzing and reasoning; creating; integrating; and making real-world connection. "Our findings suggest that if teachers purposefully and persistently practice higher order thinking strategies for example dealing in class with real-world problems, encouraging open-ended class discussions, and fostering inquiry-oriented experiments, there is a good chance for a consequent development of critical thinking capabilities" (Miri, David, & Uri, 2007, p. 353).

Having students engage in problem solving before direct instruction and learn from their failed problem solving attempts has been linked to significantly greater conceptual understanding as well as transfer of knowledge to novel problems (Kapur, 2014). Mueller and colleagues (2014) studied specific teacher actions that encouraged students to take responsibility for their mathematical problem solving and assume roles that might otherwise be expected as the teacher's responsibility, such as determining if solutions to a problem are correct, evaluating the reasonableness of arguments, and posing questions. "Rather than correcting students' errors, the teachers charged the students with considering the reasonableness of solutions. Students were not praised for correct solutions; rather, all solutions were considered and students were afforded the opportunity to defend and/or modify their arguments. A result was that the learners were comfortable judging their own solutions and those of their peers, and learned that they could determine the validity of a mathematical argument" (p. 16-17).

In the elementary grades, students must develop understanding and make use of the big ideas in mathematics and problem-solving tasks in ways that also contribute to understanding of those big ideas. Mathematics learning requires students to use specific mathematical reasoning processes, also known as 'big ideas,' across domains. These big ideas constitute overarching concepts that connect multiple concepts, procedures, or problems within or across domains or topics. They also serve as an important aspect of the process of forming connections and acquiring background knowledge that can be applied to expand later understanding (Cross et al., 2009).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Building reasoning and strategic problem-solving skills and providing ongoing opportunities for application are central aims of ***HMH Into Math***. The program builds understanding along progressions and big ideas that are clearly identified and carefully sequenced.

TEACHING FOR SUCCESS

TEACHING FOR DEPTH: Represent Numbers to 5 with Objects

Make Connections Tell children that they can place up to five objects in a group and count them. Practice this skill with children before beginning the module so that they gain experience with the numbers before they are asked to represent numbers up to five.

Represent and Explain Once children have practiced counting groups up to five, show them how they can make their own groups of this number.

Say, "I can make a group of five." Count out counters for children to see, counting on each one aloud until you've reached five.

Mathematical Progressions

Prior Learning	Current Development	Future Connections
Children: • used counting and numbers to determine quantities up to five.	Children: • count objects, saying the number names in standard order. • pair each object with only one number name and each number name with only one object. • understand that the last number said tells the number of objects counted. • count to answer "how many?" questions. • count out a number of objects to match a given number.	Children: • will count to 120, starting at any number less than 120. • will read and write numerals and rep a number of objects with a written numeral.

HMH Into Math embeds learning within students' background knowledge as well as within STEM connections and real-world contexts.

Learning Tasks within each lesson include the following stages in the process of developing students' reasoning and problem solving proficiency:

- **Spark Your Learning** tasks promote conceptual understanding. During these low floor/high ceiling tasks, students select manipulatives or representations that serve as their entry point. Teachers provide just-in-time support, helping students engage in meaningful discourse and learn to persevere. Teachers then lead the class to conceptual understanding by selecting students to share their solutions and discuss their mathematical reasoning.

- **Build Understanding** tasks are learning opportunities designed to help students understand lesson concepts. Teachers take a more active role, guiding discussion during whole-class instruction.

- **Step It Out** tasks build upon students' conceptual understanding to promote procedural understanding and fluency.

  Teachers help students understand why the procedures are efficient and how they can be applied to solve similar problem types.

To help teachers embrace best practices and become comfortable facilitating **Spark Your Learning** tasks, editable **Spark PowerPoint** files are included the Resources on ***Ed***. Teachers can use these classroom presentations to propel conversation forward, get students unstuck, and show student work samples with correct representations and answers.

Spark Your Learning

These salsas are made for a competition.

On Fire!

Blazin'

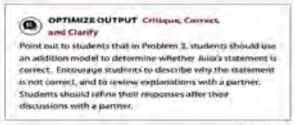
Kickin'

Feel the Burn

If peppers and tomatoes are the only ingredients in the salsas and the names are based on taste, which two salsas should have the same name? Show your work and explain.

To succeed in mathematics, students need a clear, articulated path for learning. [m]athematics instruction—like any good instruction—must be intentionally designed and carefully orchestrated in the classroom, and should always focus on impacting student learning" (hattie, fisher, & frey, 2017, p. 3-4). A coherent math curriculum is sequentially ordered to best reflect the hierarchical and logical structures of mathematics (schmidt, wang, & mcknight, 2005). "a robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades" (nctm, 2014, p.4).

In the program's **Critique, Correct, and Clarify** feature, students correct work that is not their own with a flawed explanation, argument, or solution method and share with a partner to reflect thane refine the sample work.



OPTIMIZE OUTPUT Critique, Correct, and Clarify

Point out to students that in Problem 3, students should use an addition model to determine whether Julia's statement is correct. Encourage students to describe why the statement is not correct, and to review explanations with a partner. Students should refine their responses after their discussions with a partner.

Grade 4

MATHEMATICAL MODELS AND REPRESENTATIONS

Because mathematics entails the use of signs such as symbols and diagrams to represent abstract notions and study spatial aspects, as well as because the nature of the subject is often invisible and intangible, visualizations are integral to learning and teaching mathematics (Presmeg, 2020; Bobis & Way, 2018; Stylianou, 2011). Additionally, mathematics as a tool for understanding present and future real-world problems has led to modeling becoming an important part of preparing students for advanced study and careers (Abassian, Safi, Bush, & Bostic, 2020). A wide body of research dating back decades supports the use of physical and imagistic models, manipulatives, and other such representations in the mathematics classroom; such representations help make abstract concepts more concrete as well as aid in the internalization of procedures for problem solving, increased creativity, greater metacognition, and students' more active participation in their own learning—all of which contribute key elements for impactful mathematical exploration (Carbonneau, Marley, & Selig, 2013; Cross et al., 2009; NCTM, 2000 & 2014; NRC, 2001). The positive effects of manipulative use in math instruction extend to digital tools as well as physical objects (Bouck & Park, 2018). "For students to understand such mathematical formalisms, we must help them connect these formalisms with other forms of knowledge, including everyday experience, concrete examples, and visual representations. Such connections form a conceptual framework that holds mathematical knowledge together and facilitates its retrieval and application" (Donovan & Bransford, 2005, P. 364).

NCTM (2000) recommends that K-12 instructional programs enable all students to create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena. "Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling" (p. 67).

At every level of learning, representations in the form of images, simple drawings, graphs, and other ways to see and think about mathematical ideas can show what students know, help students explain what they know, and be the foundation for making connections and achieving a deeper understanding of mathematics. Math drawings or other visual renderings are tools for modeling, sense-making, reasoning, explaining, structuring, and generalizing. Mathematical representation is commonly thought to be a product—a picture or set of symbols students makes to demonstrate understanding; however, representation in math learning is also a critical process. Students' diagrams and symbolism evolve dynamically over the course of problem solving and aid thinking and the construction of understanding in highly personal ways (Stylianou, 2011). When students sketch or organize their mathematical thinking, they are able to explore their understanding of concepts, procedures, and processes—and communicate mathematically (Arcavi, 2003; Stylianou & Silver, 2004). Having students then participate in discussions about their representations allows for meaningful learning (Fuson & Murata, 2007).

There are some mathematical skills which may be best developed with practice in the context of a meaningful examination of patterns and strategies (Fuson, 2009). A significant research base (see, for example, Baroody, 2006; Fuson, Kalchman, & Bransford, 2005; Fuson & Murata, 2007; Russell, 2000) suggests that to develop students' fluency in procedures, teachers should support students in looking for patterns and allow students to flexibly choose among solution methods. "Research indicates that discovering patterns or relations facilitates mastery with fluency....Focusing on structure, rather than memorizing individual facts by rote, makes the learning, retention, and transfer for any large body of factual knowledge more likely" (Baroody, Bajwa, & Eiland, 2009, p. 70).

Using visual representations has shown to improve student performance in general mathematics, prealgebra, word problems, and operations (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). Mathematical representations enable teachers to explain and learners to understand situations quantitatively or geometrically as they "help to portray, clarify, or extend a mathematical idea by focusing on its essential features" (NCTM, 2000, p. 206). Representations bolster intuition and understanding (Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007) and can help students to communicate, reason, problem solve, connect, and learn (Hill, Sharma, Obyrne, & Airey, 2014). Researchers have concluded that visualization is a powerful problem-solving tool and can be helpful in all kinds of mathematical problems, not only geometric problems (Van Garderen, 2006). "[I]magery based processes play an important role in all levels of mathematical problem solving…" (Watson, Campbell, & Collis, 1996, p. 177).

An effective approach to mathematical modeling with real world application entails putting students into distinct roles in which they acquire necessary information to solve a problem as well as analyze and connect that information in order to solve problems with multiple solutions (Kaplinsky, 2018).

Visual representations, models, and manipulatives are important in math learning for all students, but a large body of research also indicates that students who have special needs or at-risk, are multilingual or those having difficulty grasping abstract mathematical concepts especially benefit from visual representations of mathematical ideas, including physical objects they use or actions they perform as they are trying to solve problems (Bouck & Park, 2018; NRC, 2001; Riccomini, Witzel, & Deshpande, 2022).

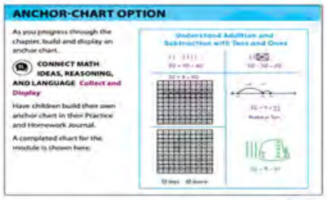
At the earliest grade levels, visual representations are particularly helpful in building students' understanding of number and geometry. Visual representations can help clarify concepts of tens and ones in the number systems—concepts that are made less clear by the structure of the English language. For young students, these visual representations and drawings of tens and ones can support understanding (Fuson, 2009). In a study examining first- and second-graders using concrete manipulatives to learn symbolic multi-digit addition and subtraction procedures, Fuson (1986) found that "for many children who made procedural errors on delayed tests, the mental representation of the procedure with the physical embodiment was strong enough for them to use it to self-correct their symbolic procedure" (p. 35). Other studies demonstrate the effectiveness of using concrete materials and pictorial representations when teaching students with learning disabilities, dyslexia, and other language difficulties; such research shows that such approaches successfully aid students with learning disabilities in mastering math concepts including algebra skills, basic math facts, coin sums, fractions, multiplication, and place value (Miller & Hudson, 2007).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math deepens student understanding with hands-on learning, including by providing a **Manipulative Kit** at - and unique to - each grade level.

The program's **Digital Toolbox** includes online manipulatives, such as base-ten blocks, students can use to produce mathematical representations to make learning more concrete. This tool also allows students to self-check their own work, fostering agency and independence.
<<Into Math G3 digital toolbox, Base-Ten Blocks: Add>>

Through **Anchor Charts**, students capture oral words and phrases learned and build a collective reference containing illustrations, concepts, and terms within each module.



ANCHOR-CHART OPTION

As you progress through the chapter, build and display an anchor chart.

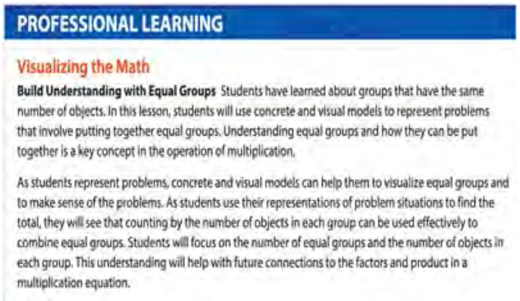
CONNECT MATH IDEAS, REASONING, AND LANGUAGE **Collect and Display**

Have children build their own anchor chart in their Practice and Homework Journal.

A completed chart for the module is shown here.

Grade 1

Professional Learning within the ***HMH Into Math*** Teacher's Edition aids support for representation and visualization.



PROFESSIONAL LEARNING

Visualizing the Math

Build Understanding with Equal Groups Students have learned about groups that have the same number of objects. In this lesson, students will use concrete and visual models to represent problems that involve putting together equal groups. Understanding equal groups and how they can be put together is a key concept in the operation of multiplication.

As students represent problems, concrete and visual models can help them to visualize equal groups and to make sense of the problems. As students use their representations of problem situations to find the total, they will see that counting by the number of objects in each group can be used effectively to combine equal groups. Students will focus on the number of equal groups and the number of objects in each group. This understanding will help with future connections to the factors and product in a multiplication equation.

MATHEMATICAL LANGUAGE AND COMMUNICATING MATHEMATICALLY

"Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and argument" (NCTM, 2014, p. 29). Research has long demonstrated that mathematical proficiency is about far more than numbers. Encouraging students to verbalize problems before giving a written response has been found to increase the rate of correct answers (Lovitt & Curtis, 1968) while encouraging students to verbalize their current understandings and providing feedback to students increases learning gains (Gersten & Chard, 2001).

Indeed, having students communicate mathematically is an essential best practice in math learning. Back in 2000, the National Council of Teachers of Mathematics adopted a Communication Standard, which notes that ""Communication is an essential part of mathematics and mathematics education...[that] can support students' learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write, and use mathematical symbols...the communication process also helps build meaning and permanence for ideas and makes them public" (p. 59-60).

As with all fields of learning, mathematics has its own language and "like all language skills, learning the language of mathematics is an important goal for all students and can remove barriers to learning mathematical ideas" (Dacey, Lynch, & Salemi, 2013, p. 149). While it is essential that students learn math-specific vocabulary, it is equally critical that students engage with that terminology and broader mathematical concepts through discourse. Mathematical discourse—speaking, writing, or listening about mathematics—is an important way for students to learn and make sense of mathematics; such communicative exchanges provide access to ideas, relationships among those ideas, strategies, procedures, facts, and mathematical history as well as foster deeper understanding and positive attitudes toward mathematics (Morgan, Craig, Schütte &, Wagner, 2014; Leinwand & Fleischman, 2004; Michaels, O'Connor, & Resnick, 2008; Smith & Stein, 2011).

"Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical for future success in mathematics" (Carpenter, Franke, & Levi, 2003, p. 4).

Discourse within mathematics learning setting, especially when marked by teachers' encouragement that students verbalize their thinking and understanding and their provision of feedback to students on that shared verbalization has been shown to benefit students across grade levels in their development of reasoning and problem-solving skills (Humphreys & Parker, 2015).

Discourse also provides teachers with opportunities for assessment. "Mathematical conversations provide opportunities for teachers to hear regularly from their students and to learn about the range of ideas students have about a particular mathematical idea, the details supporting students' ideas, the values students attach to those ideas, and the language students use to express those ideas. The knowledge teachers gain from engaging with their students in conversations is essential for teaching for understanding" (Franke, Kazemi, & Battey, 2007, p. 237).

A classroom in which meaningful communication and discussion are primary vehicles for learning and in which members co-construct and support one another's understanding is known as a "Math-Talk Learning Community;" within effective math-talk communities, teachers shift from the traditional role of directing all learning to one more like a coach or facilitator who promotes greater student agency (Hufferd-Ackles, Fuson, & Sherin, 2004 & 2015; Saylor & Walton, 2018). Math talk is an essential component of mathematical thinking and to be effective math talkers, students need to develop skills across the components of questioning, explaining mathematical thinking, identifying the source of mathematical ideas, taking responsibility for learning, and mathematical representations. (Cuoco, Goldenberg, & Mark, 1996).

"The informal and formal representations and experiences need to be continually connected in a nurturing 'math talk' learning community, which provides opportunities for all children to talk about their mathematical thinking and produce and improve their use of mathematical and ordinary language" (Cross, et al., 2009, p. 43). "Math talk" conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning (Mercer & Howe, 2012; Zwiers, 2014). The frequency of teachers' math talk has been shown to correlate with students' increased mathematical knowledge (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). "Math Talk" benefits students at different levels of learning and in different contexts, including English language learners in particular (Hufferd-Ackles, et al., 2004 & 2015). Bray and colleagues (2006) found that as students in transitional language classroom engaged in math talk, they "communicate verbally and in writing about their mathematical ideas, they not only reflect on and clarify those ideas but also begin to become a community of learners" (p. 138).

The incorporation of writing into K-12 mathematics instruction has over the past few decades become increasingly prevalent, including as a means of assessing understanding of concepts and procedures (Powell, Hebert, Cohen, Casa, & Firmender, 2017). Numerous studies demonstrate evidence that writing is also an important aspect of effective math learning across the grade span and that conceptual understanding and problem-solving skills improve when students are encouraged to write about their mathematical thinking (Bossé & Faulconer, 2008; Graham, Kiuhara, & MacKay, 2020; Russek, 1998; Wilcox & Monroe, 2011).

Writing during math instruction has been found to give students more confidence in their math abilities, create more positive attitudes toward math, and help students to understand complex math concepts and effectively problem solve (Taylor & McDonald, 2007; Williams, 2003). In a synthesis of empirical research examining 29 studies of writing used in elementary and secondary math instruction conducted between 1991 and 2015, it was concluded that writing should be implemented systematically and explicitly, with appropriate scaffolds to support the development of math communication skills (Powell et al., 2017).

Research also shows that writing during math learning has benefit for all students, both low-achieving (Baxter, Woodward, & Olson, 2005) and high-achieving (Brandenburg, 2002). Writing can be effectively incorporated into the mathematics classroom in a wide variety of ways, both formal and informal (Urquhart, 2009). Researchers also cite journal writing as having positive impacts on math achievement and affective experiences and perceptions of math learning (Page & Clarke, 2014)—including specifically in algebra classrooms, where journals were found to aid the development of reasoning, sense-making, and discourse (Yow, 2015).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math presents each and every student with frequent opportunities to speak, write, read, and listen in the mathematics classroom. ***HMH Into Math*** delivers these communication activities, which are greatly amplified through the support and encouragement provided for teachers, with strategies to produce growth for students within each language proficiency.

As shown in the Embedded Language Development and Support section above, the program was designed with four guiding principles to foster mathematical language use and development in the classroom: support sense-making; optimize output; cultivate conversation; and maximize linguistic and metacognitive awareness. The four design principles, infused with Mathematics Language Routines, are the foundation for the language development practices in ***HMH Into Math.***

The program's approach to vocabulary instruction is designed to bridge the gap between academic language and understanding, ***HMH Into Math*** allows for academic vocabulary to emerge after students explore a concept and develop understanding. Rather than being front-loaded, new vocabulary is highlighted after the concept is taught, connecting students' understanding of the concept to the explicit vocabulary term. With this unique approach, English learners simultaneously boost their disciplinary and English language abilities.

- **Key Academic Vocabulary** is labeled as Review Vocabulary from prior learning or as New Vocabulary that is currently being developed.
- **Language Objectives** are clearly stated for each lesson, developing students' meta-awareness through identifying, describing, explaining, and evaluating the mathematical content.
- An English/Spanish **Interactive Glossary** provides students with the space to make sense of key vocabulary terms with their own words and illustrations.
- A **Multilingual eGlossary** offers additional access to key terms with an interactive tool in 10 languages.
- English/Spanish **Vocabulary Cards & Games** reinforce and develop mathematical vocabulary through peer communication.
- New and **high-utility vocabulary** is highlighted at point of use in the Student Edition with support offered in the Teacher's Edition.

Key Academic Vocabulary, Grade 1

Connect to Vocabulary

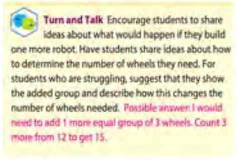
A **unit fraction** tells the part of a whole that each piece represents. The numerator of a unit fraction is always 1.

High-Utility Vocabulary, Grade 4

Interactive Glossary, Grade 1

Additionally, **Key Academic Vocabulary** is labeled as Review Vocabulary from prior learning or as New Vocabulary that is currently being developed while new and **high-utility vocabulary** is highlighted at point of use in the Student Edition with support offered in the Teacher's Edition.

The ***HMH Into Math*** Teacher's Edition establish and maintain effective math talk communities, including through its recurring **Turn and Talk** feature.



Turn and Talk Encourage students to share ideas about what would happen if they build one more robot. Have students share ideas about how to determine the number of wheels they need. For students who are struggling, suggest that they show the added group and describe how this changes the number of wheels needed. Possible answer: I would need to add 1 more equal group of 3 wheels. Count 3 more from 12 to get 15.

HMH Into Math offers a wealth of resources to support discourse and writing about math. These include **academic notebooks** and **math journals,** with accompanying **Put It in Writing** prompts that requires students to explain their understanding, as well as interactive **glossaries**. The **Learn Together** feature guides students through Build Understanding and Step It Out tasks and facilitate discourse during whole-class instruction to help students see relationships and how mathematical ideas are connected.

PURPOSEFUL QUESTIONS

"Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014, p. 35). A major factor impacting teaching and learning of mathematics is the quality of classroom discourse (Hiebert & Wearne, 1993; Smith & Stein, 2011). While many teachers of mathematics allow students to communicate their mathematical thinking, it is critical that such discussions employ effective questioning techniques that genuinely support increased understanding (Childs & Glenn-White, 2018). Developing appropriate questioning techniques is such an important part of mathematics teaching and assessment that one study found "[a] good question may mean the difference between constraining thinking and encouraging new ideas, and between recalling trivial facts and constructing meaning" (Moyer & Milewicz, 2002, p. 293).

Teachers' questions are crucial in helping students make connections and learn important mathematics concepts. Questions are a means of both fostering understandings and evaluating understandings (Hattie et al., 2017)—particularly in the early grades (Stiles, 2016). Questioning techniques shape learning experiences in significant ways, including how students see themselves and their capabilities (Goffney, 2018). Classroom discussions should be organized in ways that have been shown to support the acquisition of mathematics concepts and language development (Smith & Stein, 2018). Asking "why?" and "how do you know?" is one strategy that effective teachers use to encourage students to explain their thinking, solve problems, and share mathematical strategies and ideas with their peers (Clements & Sarama, 2004 & 2007). Without expert guidance, discussions in mathematics classrooms can easily devolve into the teacher taking over the lesson and providing a "lecture," on the one hand, or, on the other the students presenting an unconnected series of show-and-tell demonstrations" (Smith & Stein, 2018, p. 4). The kinds of questions math teachers ask and the kind of support that teachers offer are critical on an affective as well as cognitive level, as questions may either facilitate or undermine students' productive efforts and determine whether students view struggle as a positive endeavor or the source of difficulty and frustration (Warshauer, 2015).

In *Essential Questions: Opening Doors to Student Understanding* (2013), McTighe and Wiggins explain that questions are important for stimulating student thinking and inquiry as well as for helping teachers target standards and other goals for learning outcomes. Essential questions should develop and deepen students' understanding of important ideas and processes so that students can transfer their learning within and outside school. McTighe and Wiggins suggest that content be unpacked to identify long-term transfer goals and desired understandings in a process that entails the development of associated essential questions. In other words, essential questions can be used to effectively frame key learning goals. Targeted understandings and essential questions are intrinsically related. In addition to addressing goals for learning, essential questions can be characterized as provocative and generative; they are open-ended, thought provoking, require higher order thinking, spark additional questions; point toward larger, transferable ideas; require justification; and recur across the curriculum.

NCTM (2014) recommends that teachers present questions drawing from a research-based framework of types that include the following categories:

- gathering information: recall of facts, definitions, and procedures
- problem thinking: explain, elaborate, or clarify thoughts, including the articulation of steps in solution method or task completion
- making learning visible: discuss mathematical structures and make connections among mathematical ideas and relationships
- reflection and justification: reveal deeper understanding of reasoning and actions, including making arguments for validity of their work.

It is also important, NCTM points out, that in addition to a variety of question times, teachers employ patterns of questioning, including allotting sufficient response time, that focus on and extend students' current ideas to advance their understanding and sense-making about essential mathematical ideas and relationships.

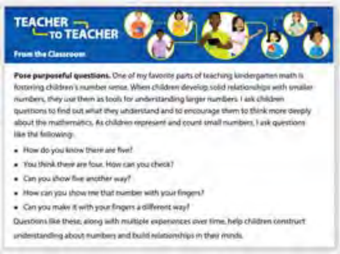
"[T]he key to purposeful questioning is intentionality. Purposeful questioning will not occur through happenstance. These questions will help guide the unpacking of the mathematical task during individual, small-group, and whole- class discussion. It is imperative to incorporate questions that lead discussions to move beyond focusing on students merely obtaining the correct answer, to discussions that focus on making sense of the problem-solving process. Focusing questions on the problem-solving process helps to enrich student's mathematical experiences by allowing the mathematics to move beyond just 'numbers' and 'formulas' to a beautiful concept that is based upon problem-solving" (Childs & Glenn-White, 2018, p. 15-16).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math supports students' mathematical practices and processes through strategic question posing plus an abundant support for teachers in developing effective, purposeful questioning techniques. Following is a summary of embedded teacher questioning guidance.

MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM.	• What is the problem asking? • How will you use that information? • What other information do you need? • What is another way to solve that problem? • What can you do if you don't know how to solve a problem? • Have you solved a problem similar to this one? • How do you know your answer makes sense?
REASON ABSTRACTLY AND QUANTITATIVELY.	• What quantities are referenced? • How are the quantities related? • How can you represent this situation? • How do the quantities and the units relate? • What are the correct units for the quantities in the problem? • How do you know your answer is reasonable?
CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS.	• Will that method always work? How do you know? • What do you think about what the other student said? • Who agrees or disagrees, and why? • Does anyone have another way of looking at that? • What do you think will happen if...? • When would that not be true? • Does that make sense to you? Why?
MODEL WITH MATHEMATICS.	• Why is that a good model for this problem? • How can you use a simpler problem to help you find the answer? • What conclusions can you make from your model? • Do your results make sense with the context of the problem? • How would you change your model if...?
USE APPROPRIATE TOOLS STRATEGICALLY.	• What could you use to help you solve the problem? • What strategy could you use to make that calculation easier? • How would estimation help you solve that problem? • Why did you decide to use...?
ATTEND TO PRECISION.	• How do you know your answer is reasonable? • How can you use math vocabulary in your explanation? • How do you know those answers are equivalent? • What does that mean?
LOOK FOR AND MAKE USE OF STRUCTURE.	• What rule did you use to make this group? • Why can you use that property in this problem? • How is that like...?
LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING.	• How did you discover that pattern? • What other patterns can you find? • What do you remember about...? • What happens when...? • What if you...instead of...? • What might be a shortcut for...?

The *HMH Into Math* Teacher's Edition also provides ongoing guidance for posing effective, purposeful questions, such as via the **Teacher-to-Teacher** feature.



TEACHER TO TEACHER

From the Classroom

Pose purposeful questions. One of my favorite parts of teaching kindergarten math is fostering children's number sense. When children develop solid relationships with smaller numbers, they use them as tools for understanding larger numbers. I ask children questions to find out what they understand and to encourage them to think more deeply about the mathematics. As children represent and count small numbers, I ask questions like the following:

- How do you know there are five?
- You think there are four. How can you check?
- Can you show five another way?
- How can you show me that number with your fingers?
- Can you make it with your fingers a different way?

Questions like these, along with multiple experiences over time, help children construct understanding about numbers and build relationships in their minds.

CONCEPTUAL UNDERSTANDING AND PROCEDURAL FLUENCY

Researchers and experts have identified the importance of an integrative approach to mathematics instruction that focuses on and balances both conceptual understanding and procedural fluency (National Mathematics Advisory Panel, 2008; NRC, 2001; NCTM, 2014). Conceptual understanding is knowledge of abstract and general principles whereas procedural understanding is knowledge of the steps or actions between a goal that is then applied with varying degrees of fluency (Rittle-Johnson, Schneider, & Star, 2015). "Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems" (NCTM, 2014, p. 42).

"Procedural knowledge and conceptual understandings must be closely linked" (NRC, 2005, p. 232) and effective mathematics cannot have one without the other, for concepts and procedures develop in tandem and iteratively, with gains in one supporting gains in the other (NCTM, 2014; Rittle-Johnson et al., 2015). As charted by Larson and Kanold (2016), among math educators in the United States, a longstanding tension has existed between understanding and fluency; on one side is an emphasis on exploration facilitated by sensory experiences with objects and open-ended activity and on the other is a focus on rote practice and worksheets without attention to the construction of meaning.

However, as Fuson (2009) urges in addressing this divide, students learning math are best served by a balanced approach that is child-centered and a structure that is teacher-guided, where individualized pathways driven by each student's needs and progress receive a dual focus on both understanding and fluency. This approach helps avoid mathematical teaching without learning, in which rote practice and worksheets are utilized without a focus on meaning-making.

Conceptual understanding benefits students because it allows them to make connections between current knowledge and new topics and thereby learn more quickly. Additionally, conceptual understanding helps students avoid critical errors because they can readily assess the reasonableness of solutions (NRC, 2001). "Judging the reasonableness of computational results is pivotal for students to understand mathematical concepts. This domain is the most sensitive to the presence of misconceptions in mathematics" (Yang & Sianturi, 2019, abstract).

Procedural fluency is a critical component of mathematical proficiency. More than memorizing facts or steps, it entails the following capacities: to apply procedures accurately, efficiently, and flexibly; to build and modify procedures as well as transfer them to different problems and contexts; and to recognize when one strategy or procedure is more appropriate to apply than another. In developing procedural fluency, students need experience integrating concepts and procedures and understanding patterns among them. Students also need to build on familiar procedures in the process of creating their own informal strategies and procedures via opportunities to support and justify their choices of appropriate procedures.

To be mathematically proficient, students need a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations—and the goal for students developing procedural fluency is to acquire a body of known facts and generalizable methods that will allow them to efficiently and accurately solve varied problems (NRC, 2001 & 2005). Finally, to strengthen their understanding and skill of procedural fluency, students need consistent, meaningful, engaging, purposeful—decidedly not rote—practice that is distributed over time (Baroody et al., 2009; Fuson & Murata, 2007; NCTM, 2015; Rohrer, 2009).

When students are able to connect procedures and concepts, when learning is meaningful, retention improves and students are better able to apply what they know in different situations. If students memorize and practice procedures without conceptual understanding, they lack capacity to apply procedures and the motivation to used them effectively (Fuson, Kalchman, & Bransford, 2005; Hiebert, 1999). A strong evidence base (see, for example, Baroody, 2006; Fuson & Beckmann, 2012/2013; Fuson, et al., 2005; Fuson & Murata, 2007; Russell, 2000) suggests that to develop students' fluency in procedures, teachers should: build on a foundation of conceptual understanding; support students in looking for patterns; allow students to flexibly choose among solution methods; and offer distributed opportunities for purposeful, meaningful practice (not rote, repeated practice). Practice is indeed key to developing procedural fluency. Students should have opportunities for practice that are brief, engaging, purposeful, and distributed over time (Rohrer, 2009).

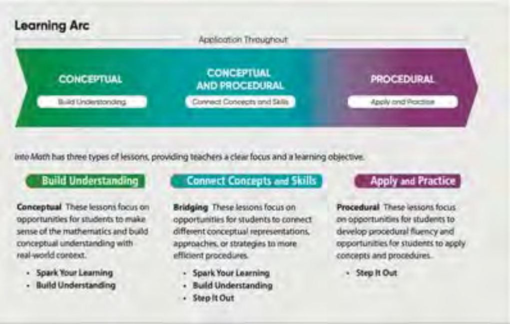
Rittle-Johnson (2017) recommends three specific cognitive activities within learning tasks that promote the development of conceptual knowledge, procedural knowledge, and procedural flexibility simultaneously. One of these is comparison: comparing alternative processes for solving a problem as well as comparing correct versus incorrect procedures in solving a problem. Another is self-explaining: generating explanations to make sense of new information as well as explanations of solutions to math problems via in part by connecting new information and explanation to background knowledge. Third is exploration before instruction. Students who have opportunity to solve an unfamiliar problem or devise their own formulas and approaches to an unfamiliar problem in advance of receiving teacher-directed instruction typically demonstrate more positive gains and outcomes.

For students to build conceptual understanding and procedural fluency, they must extend their new knowledge and skill into application via processes that also allow students to demonstrate strategic competence and adaptive reasoning (NRC, 2001). Additionally, correctly applying mathematical knowledge depends on solid conceptual knowledge and procedural fluency. Any meaningful application of mathematical knowledge draws on both conceptual understanding and procedural fluency and provides a real-world, problem-based context (David & Greene, 2007; Cross et al., 2009; Gaddy, Harmon, Barlow, Milligan, & Huang, 2014; Hiebert et al., 1996; NCTM, 2014). If students attempt to begin solving real-world problems while lacking knowledge and fluency, problems are made unnecessarily, perhaps prohibitively, challenging. Yet at the same time, educators should not save all application for the end of learning progressions. Application can be motivational and interesting, and students at all levels need to connect the mathematics they are learning to the world around them (Alberti, 2013).

HOW *HMH INTO MATH ALIGNS* TO RESEARCH

Today's standards require a focused, coherent, and rigorous curriculum to ensure students develop an in-depth understanding of mathematical concepts and language. Rigorous instruction must include a balanced approach, giving equal emphasis to conceptual understanding, procedural skill and fluency, and application.

Lessons in *HMH Into Math* form a coherent sequence called a Learning Arc, designed to build a foundation of conceptual understanding before teaching procedures. Opportunities for application are found throughout. An emphasis is placed on connections between concepts and skills. The Learning Arc ensures delivery of rigorous instruction.



Into Math has three types of lessons, providing teachers a clear focus and a learning objective.

Build Understanding

Conceptual These lessons focus on opportunities for students to make sense of the mathematics and build conceptual understanding with real-world context.

- **Spark Your Learning**
- **Build Understanding**

Connect Concepts and Skills

Bridging These lessons focus on opportunities for students to connect different conceptual representations, approaches, or strategies to more efficient procedures.

- **Spark Your Learning**
- **Build Understanding**
- **Step It Out**

Apply and Practice

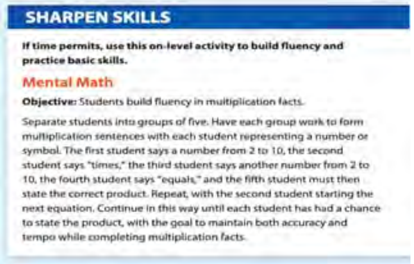
Procedural These lessons focus on opportunities for students to develop procedural fluency and opportunities for students to apply concepts and procedures.

- **Step It Out**

HMH Into Math **Learning Tasks** carry out an integrative, iterative approach to developing students' conceptual understanding and procedural fluency:

- **Spark Your Learning** tasks promote conceptual understanding. During these low floor/high ceiling tasks, students select manipulatives or representations that serve as their entry point. Teachers provide just-in-time support, helping students engage in meaningful discourse and learn to persevere. Teachers then lead the class to conceptual understanding by selecting students to share their solutions and discuss their mathematical reasoning.
- **Build Understanding** tasks are learning opportunities designed to help students understand lesson concepts. Teachers take a more active role, guiding discussion during whole-class instruction.
- **Step It Out** tasks build upon students' conceptual understanding to promote procedural understanding and fluency. Teachers help students understand why the procedures are efficient and how they can be applied to solve similar problem types.

The *HMH Into Math* **Teacher's Edition** provides additional, ongoing guidance for developing math concepts and fluency. <



SHARPEN SKILLS

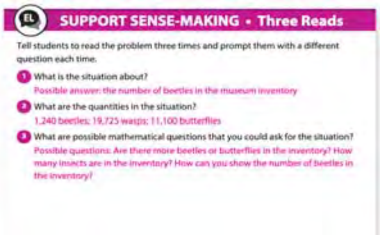
If time permits, use this on-level activity to build fluency and practice basic skills.

Mental Math

Objective: Students build fluency in multiplication facts.

Separate students into groups of five. Have each group work to form multiplication sentences with each student representing a number or symbol. The first student says a number from 2 to 10, the second student says "times," the third student says another number from 2 to 10, the fourth student says "equals," and the fifth student must then state the correct product. Repeat, with the second student starting the next equation. Continue in this way until each student has had a chance to state the product, with the goal to maintain both accuracy and tempo while completing multiplication facts.

The program's **Support Sense-Making** feature fosters the development of conceptual understanding for all students while also providing language development support, particularly for English Learners.



EL **SUPPORT SENSE-MAKING • Three Reads**

Tell students to read the problem three times and prompt them with a different question each time.

① What is the situation about?
Possible answer: the number of beetles in the museum inventory
② What are the quantities in the situation?
1,240 beetles; 19,725 wasps; 11,100 butterflies
③ What are possible mathematical questions that you could ask for the situation?
Possible questions: Are there more beetles or butterflies in the inventory? How many insects are in the inventory? How can you show the number of beetles in the inventory?

PRODUCTIVE PERSEVERANCE

Success in mathematics depends on at least two common components: practice and perseverance (Larson, 2016). "An effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics" (NCTM, 2014, p. 11). Through productive perseverance, students grapple with the issues and are able to find solutions on their own, allowing them to persist and build resilience as they pursue learning and understanding (Jackson & Lambert, 2010), realizing that through effort and tenacity alongside sense making and problem solving, they are capable of doing well in mathematics (NCTM, 2014).

To cultivate mathematical habits of mind, teachers also must create a classroom culture that demonstrates how challenge is a natural part of the learning process allows students to see the benefits of perseverance, and provides specific descriptive feedback to students on their progress related to their efforts (Hattie & Timperley, 2007; Star, 2015). This attitudinal state with regards to challenge yields numerous positive affective outcomes and boosts academic achievement (Dweck, 2006, 2008, & 2015; Hiebert & Grouws, 2007; NCTM, 2014).

Research shows that productive perseverance is necessary to the process of learning mathematics with understanding. When students are given opportunity to grapple with ideas, make mistakes, persist through difficulties, and arrive at solutions, learning outcomes improve (Hiebert & Grouws, 2007; Kapur, 2014; Warshauer, 2015). It has also been found that students given time to make mistakes and persist through their struggles ultimately show greater understanding on posttest measures than their counterparts (Kapur, 2010). Perseverance through problem solving also encourages students to think about their own thinking and to discover that authentic learning happens without rushing to simply find the correct answer (Hiebert & Grouws, 2007). "Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics" (NRC, 2001, p. 131).

To effectively foster students' productive dispositions, teachers must carefully select tasks and provide reassurance and guidance that students need to complete the tasks—but without diminishing the cognitive demand of the task or giving students too much help or direct answers. Students need sufficient time, not only to persist through challenging and devise solutions, but also to develop curiosity and stamina (Goldenberg, Mark, Kang, Fries, Carter, & Cordner, 2015; Pascale, 2016). The kinds of questions teachers ask and the kind of support that teachers offer are critical, as they either facilitate or undermine the productive efforts of students' struggles and determine whether students view struggle as a positive endeavor or the source of difficulty and frustration (Warshauer, 2015). Timing of support also plays a vital role. When scaffolding is given to students before they have the opportunity to make sense of a challenging task independently, they are inhibited in the process of developing productive perseverance. "All too often, so much support is provided through the initial scaffolding that the cognitive demand of the task is significantly decreased (Boston & Wilhelm, 2015). If this sort of scaffolding is provided upfront for students who struggle, then these same students are denied access to cognitively demanding tasks. When access is denied, equity becomes an issue" (Dixon, 2018, online).

Other practices that support productive perseverance include heterogeneous grouping, effective teacher-directed questioning, setting problems in a setting familiar to students and that draws from their everyday lives, plus goal setting before and reflection after problem solving (Pascale, 2016) plus "low floor/high ceiling" learning tasks (Sircar & Titus, 2015).

Productive perseverance makes important contributions in the promotion of a growth mindset. A growth mindset within mathematics emphasizes teaching and learning as processes that cultivate mathematical abilities; stresses that success and learning are reflections of effort and not intelligence; and promotes a belief that all students are capable of participating and achieving in mathematics. Society has traditionally valued the math learner who can memorize well and calculate fast, rather than others who possess equal potential but may be deeper, slower, and possibly more creative. These earlier mindsets have contributed to persistent negative perceptions within mathematics education specifically—and the evolution toward pervasive growth mindsets are necessary if all students are to be successful math learners.

Teachers should foster and display a growth mindset by valuing all students' thinking and efforts while also relying on pedagogical practices such as differentiated tasks, mixed-ability groupings, and praise for students' contributions and perseverance within their mathematical learning (Dweck, 2006, 2008, & 2015; NCTM, 2014). Schools and classrooms that reinforce growth mindset messaging make learning enjoyable and place the focus on that learning rather than on students' performance (Yeager, Walton, & Cohen, 2013). Setting and supporting rigorous expectations and a genuine belief that student effort and effective instruction outweigh "smarts" and life circumstances increase students' opportunities to learn—and create a more equitable learning experience (NCTM, 2014).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math provides ongoing opportunities for students to persevere in their learning productively and, when and if needed, with appropriate scaffolding. **Spark Your Learning** activities engage students in a productive perseverance task in which they explain mathematical ideas and reason about mathematical relationships. Accompanying Teacher Edition content supports teachers in guiding process with effective questioning and scaffolding to ensure positive cognitive and affective outcomes—and increased understanding.

① **Spark Your Learning**

▶ **MOTIVATE**

Introduce the problem. **Ask students:** What do you know about robots? What kinds of robots have you seen? Discuss how a robot might be built.

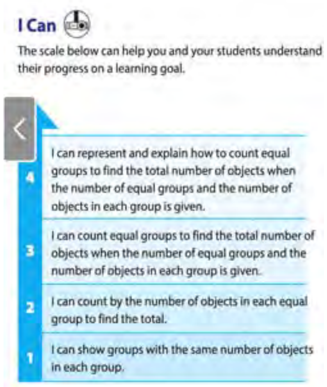
SUPPORT SENSE-MAKING **Three Reads** Have students read the problem three times. Use the questions in the Three Reads box below for a different focus each time.

▶ **PERSEVERE**

If students need support, guide them by asking:

- **Assessing** How do you know how many wheels to u for one robot? Possible answer: The picture shows th the robot has 3 wheels. This tells how many wheels to use for one robot.
- **Assessing** Would you change the number of wheels for each robot or use the same number of wheels? Why? Possible answer: The number of wheels for each robot is the same because the problem says, "you are building more robots like this one."
- **Advancing** • **Use Tools** Which tool could you use to solve the problem? Why is this tool more strategic? Students' choices of strategies or tools will vary.
- **Advancing** What methods do you know for finding the total number of objects? Possible answers: I can count to find the total number of objects. I can add to find the total number of objects.

I Can statements for students aid students in seeing themselves as capable, persistent learnings. Accompanying I Can Scales provide support in self-assessment and reflection.

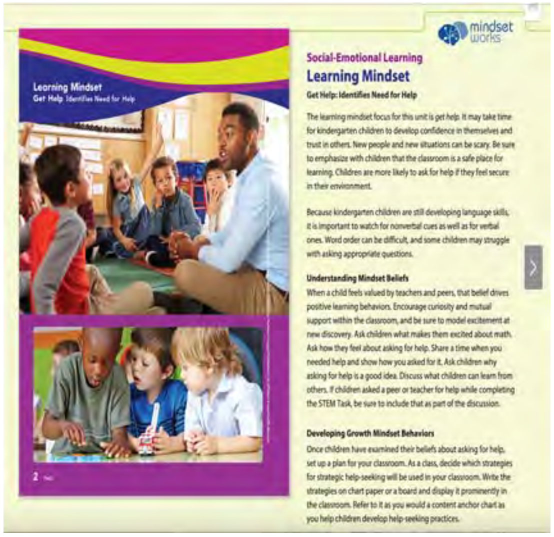


I Can

The scale below can help you and your students understand their progress on a learning goal.

4	I can represent and explain how to count equal groups to find the total number of objects when the number of equal groups and the number of objects in each group is given.
3	I can count equal groups to find the total number of objects when the number of equal groups and the number of objects in each group is given.
2	I can count by the number of objects in each equal group to find the total.
1	I can show groups with the same number of objects in each group.

HMH Into Math supports teachers in fostering positive a learning mindset for students that encourages them to persist through challenging content and tasks and perceive themselves as capable learners.



Social-Emotional Learning

Learning Mindset

Get Help: Identifies Need for Help

The learning mindset focus for this unit is get help. It may take time for kindergarten children to develop confidence in themselves and trust in others. New people and new situations can be scary. Be sure to emphasize with children that the classroom is a safe place for learning. Children are more likely to ask for help if they feel secure in their environment.

Because kindergarten children are still developing language skills, it is important to watch for nonverbal cues as well as for verbal ones. Word order can be difficult, and some children may struggle with asking appropriate questions.

Understanding Mindset Beliefs

When a child feels valued by teachers and peers, that belief drives positive learning behaviors. Encourage curiosity and mutual support within the classroom, and be sure to model excitement at new discovery. Ask children what makes them excited about math. Ask how they feel about asking for help. Share a time when you needed help and show how you asked for it. Ask children why asking for help is a good idea. Discuss what children can learn from others. If children asked a peer or teacher for help while completing the STEM Task, be sure to include that as part of the discussion.

Developing Growth Mindset Behaviors

Once children have examined their beliefs about asking for help, set up a plan for your classroom. As a class, decide which strategies for strategic help-seeking will be used in your classroom. Write the strategies on chart paper or a board and display it prominently in the classroom. Refer to it as you would a content anchor chart as you help children develop help-seeking practices.

EVIDENCE OF STUDENT THINKING

Research is increasingly finding that, particularly in science and math, teachers' engagement with student thinking is critical for supporting student learning (Black, Harrison, Lee, Marshall, & Wiliam, 2003; Dyer & Sherin, 2015) and especially when part of a larger responsive approach in which teachers use evidence of student thinking to infer and adapt instructional objectives (Hammer, Goldberg, & Fargason, 2012). To discover what students know or don't know, what they do well or poorly, the teacher must closely examine students' work. "Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (NCTM, 2014, p. 53).

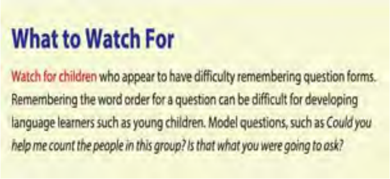
A focus on an evidence approach entails specificity and intentionality (NCTM, 2014) and it is a critical component of effective, systematic formative assessment (Wiliam, 2011). The approach begins with a clear understanding of what constitutes indication of students' mathematical thinking and what is important to notice about it as well as planning ahead of each lesson for ways to elicit that information, via deliberate questioning that reaches every student during and after the lesson. Then, once the information has been elicited, it is also necessary to interpret what the evidence means with respect to learning goals and decide how to respond on the basis of student understanding and progress toward those goals (Chamberlin 2005; Jacobs, Lamb, & Philipp 2010; Leahy et al., 2005; NCTM, 2014; Sherin & van Es, 2003).

Evidence of student thinking takes a variety of forms, such as verbal responses and gestures as well as written ones. Good sources for identifying indicators of student thinking is a math curriculum's learning trajectories describing how students' understanding develops over time (Clements & Sarama, 2004; Sztajn et al., 2012). It is important that the eliciting of thinking happens strategically, via deliberate questions aimed at identifying specific understandings and conceptual gaps as well as consideration of common patterns of reasoning that are revealed in a student's thinking, which include difficulties, errors, and misconceptions (Bray, 2013). To be effective, evidence gathering and subsequent responsive action must happen while learning unfolds and before remediation becomes necessary (Heritage, 2008; Leahy et al., 2005; NCTM, 2014).

The process of using evidence of student thinking to guide instruction necessarily includes teacher feedback. Supportive responses from teachers include asking students to restate problems in their own words, reminding them of available strategies or tools, or to change a problem to easier numbers. Extending responses have students use advanced strategies to solve the same or similar problem or have students compare and contrast strategies in selection which to apply. While there is no one-size-fits-all way to respond, the aim should always be to foster greater conceptual understanding and procedural fluency (Jacobs & Ambrose, 2008; NCTM, 2014). To be meaningful and impactful, the teacher feedback itself needs to cause thinking; while grades, scores, and comments like "good job" don't generate student thinking, what does is reference to a rubric when appropriate or a response that addresses specifically what a student needs to do to improve (Leahy et al., 2005).

HOW HMH INTO MATH ALIGNS TO RESEARCH

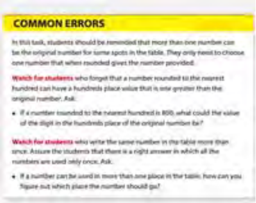
HMH Into Math supports teachers in identifying opportunities to use evidence of student thinking and interpreting that evidence, such as through the **What to Watch For** Teacher Edition feature.



What to Watch For

Watch for children who appear to have difficulty remembering question forms. Remembering the word order for a question can be difficult for developing language learners such as young children. Model questions, such as *Could you help me count the people in this group? Is that what you were going to ask?*

The **Teacher's Edition** provides sample student work plus guidance on how to identify and remedy common mathematical errors as well as support understanding within specific learning scenarios.



COMMON ERRORS

In this task, students should be reminded that more than one number can be the original number for some spots in the table. They only need to choose one number that when rounded gives the number provided.

Watch for students who forget that a number rounded to the nearest hundred can have a hundreds place value that is one greater than the original number. Ask:

- If a number rounded to the nearest hundred is 800, what could the value of the digit in the hundreds place of the original number be?

Watch for students who write the same number in the table more than once. Assure the students that there is a right answer in which all the numbers are used only once. Ask:

- If a number can be used in more than one place in the table, how can you figure out which place the number should go?

In the program's **Critique, Correct, and Clarify** feature, students correct work that is not their own with a flawed explanation, argument, or solution method and share with a partner to reflect then refine the sample work.

SUPPORTING ALL LEARNERS

"[W]e embrace a perspective on equity that supports teaching practices and reflective tools focused on empowerment of the whole child...All students, in light of their humanity—their personal experiences, backgrounds, histories, languages, and physical and emotional well-being—must have the opportunity and support to learn rich mathematics that fosters meaning making, empowers decision making, and critiques, challenges, and transforms inequities and injustices. Equity does not mean that every student should receive identical instruction. Instead, equity demands that responsive accommodations be made as needed to promote equitable access, attainment, and advancement in mathematics education for each student" (Aguirre, Mayfield-Ingram, & Martin, 2013, p. 9).

Early experiences with mathematics yield effects well beyond classrooms, with consequences affecting economic prosperity, well-being, and quality of life. While mathematics achievement on every scale requires that all students be expected to meet rigorous standards, each student comes to school with a unique background, skill set, perspective, strengths, and needs—and therefore must receive effective, individualized support to realize and enjoy success in math learning (Clements & Sarama, 2020; NCTM, 2014; National Mathematics Advisory Panel, 2008; Shapka, Doemene, & Keating, 2006). All children, including those from historically underserved populations are capable of learning and performing in math at high levels and a large body of research has documented that significant positive outcomes that are possible when schools and teachers address issues of equity and access (Gutiérrez, 2013; Kisker, Lipka, Adams, Rickard, Andrew-Ihrke, Yanez, & Millard, 2012; Lawrence-Brown, 2004; Lipka Sharp, Adams, & Sharp, 2007; McKenzie, Skrla, Scheurich, Rice, & Hawes, 2011). "Providing young children with extensive, high-quality early mathematics instruction can serve as a sound foundation for later learning in mathematics and contribute to addressing long-term systematic inequities in educational outcomes" (Cross, et al., 2009, p. 2).

HMH *Into Math* supports students equitably and effectively by providing access to highest quality mathematics instruction with embedded differentiation to meet wide-ranging needs. *HMH Into Math* also supports teachers by providing tools to help create nurturing classroom environments that facilitate deep learning of mathematics for all.

PROMOTING ACCESSS, EQUITY AND RIGOR FOR ALL LEARNERS

For over two decades, the National Council of Teachers of Mathematics has advocated for more equitable practices that ensure all students succeed in learning math as well as for recognition that equity requires diversity of support. From NCTM's *Principles & Standards for School Mathematics*, (2000). "All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics" (p. 12).

"An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" NCTM, 2014. p. 59). Despite continually growing demands for a STEM-trained workforce (Langdon et al., 2011) as well as shrinking achievement gaps, historically underrepresented groups that include females, African American, Latinx, Native American, English learners (ELs), students in poverty and those with disabilities remain marginalized in STEM education and professions, including specifically in mathematics (Anwar, Bascou, Menekse, & Kardgar 2019; Jackson, Mohr-Schroeder, Bush, Maiorca, Roberts, Yost, & Fowler, 2021; Kang, Barton, Tan, Simpkins, Rhee, & Turner, 2019; Sneider & Ravel, 2021).

American classrooms today are increasingly diverse; individual students have wide-ranging needs but they are also best served when their own experiences and backgrounds are valued and leveraged in the course their learning experiences. All students need to learn mathematics and, with appropriate, differentiated support, all students are capable of success in mathematics. It is vital that educators understand that achievement gaps are not caused by factors such as cultural differences, poverty, and parental education levels, but rather by pervasive inequalities that have historically afforded significantly fewer resources and opportunities to certain groups (Aguirre, et al. 2013; Cross et al., 2009; Flores, 2007; Gutiérrez, 2013; Lawrence-Brown, 2004; NCTM, 2014; Ukpokodu, 2011). "Acknowledging and addressing factors that contribute to differential outcomes among groups of students are critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful" (NCTM, 2022, online).

Larson (2018) emphasizes that while improved access to quality mathematics instruction and college pathways remains essential, research indicates that other critical factors must also be part of efforts to remedy issues of equity. These include: fostering positive mathematical experiences and identities that empower all students; cultivating equitable mathematical discussions within classrooms; and inspiring today's youth to embrace and engage with a math-centric future.

In addressing issues of equity and access, calls are increasing for educators to shift away from perceptions that students from historically disadvantaged backgrounds are deficient; rather, educators are encouraged to adopt a culturally responsive approach in which the distinct cultural, linguistic, and environmental experiences and environmental students bring to school are viewed as assets to be respected, embraced, and leveraged to optimize learning for individual students as well as their peers (Aguirre, et al. 2013; Flores, 2007; Gutiérrez, 2013; Lawrence-Brown, 2004; NCTM, 2000, 2014 & 2019; Ukpokodu, 2011; Xenofontos, 2019. "[M]any of the critical challenges facing racial and ethnic minority students in the formation of strong, positive mindsets for academic achievement can be alleviated through the careful work of creating supportive contexts that provide consistent and unambiguous messages about minority students' belonging, capability, and value in classrooms and schools" (Farrington, Roderick, Allensworth, Nagaoka, Keyes, Johnson, & Beechum, 2012, p. 34).

Research (see NCTM, 2019, Xenofontoas, 2019 and others, including those cited above) suggest the following equitable, culturally responsive teaching practices:

- setting clear, rigorous expectations for all learners while also attending to each students' distinct cultural, cognitive, emotional, psychological well-being and needs;
- providing a range of high-quality, effective, and equitably distributed resources to support students.
- drawing on students' unique funds of knowledge, recognizing diverse forms of culture, perspectives, language, and discourse are assets for learning and within a classroom environment;
- allowing adequate time for students to learn;

- establishing protocols and norms for broad participation in individual classroom activities and the learning process as a whole;
- implementing differentiated processes for instruction that foster students' mathematical thinking and broaden students' productive engagement with mathematics in ways that also support individual students as needed, meeting them at their developmental level with a positive, appropriate level of challenge;
- positioning students as capable, defiant of stereotypes, and agents in their own learning and building a classroom culture in which students view their peers that way;
- attending to race and culture and other differences and experiences; and
- monitoring student progress through fair and accurate assessment and making needed accommodations accordingly; and

Additionally, to create an environment in which the barriers that limit comprehensive student access to learning are removed, teaching allows for flexible methods of presentation, expression, and engagement by offering multiple examples, employing multiple media and formats, engaging in supported practices, and allowing flexible opportunities for demonstrating skill (Strangman, Hall, & Meyer, 2004).

Attending to access and equity also means recognizing that inequitable learning opportunities can exist in any setting, diverse or homogenous, whenever only some, but not all, teachers implement rigorous curricula and equitable teaching practices that support all students. "Abundant research has documented the significant outcomes that are possible when schools and teachers systematically address obstacles to success in mathematics for students from historically underserved populations...The question is not whether all students can succeed in mathematics but whether the adults organizing mathematics learning opportunities can alter traditional beliefs and practices to promote success for all" (NCTM, 2014, p. 60-61).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math provides each and every student with equitable access to quality, evidence-based mathematics curriculum and pedagogy. In one way that the program supports equity, all students, including and especially English learners, are empowered with frequent opportunities to speak, write, read, and listen in their classroom community. Research shows the need for a shift in when and how language learners are taught mathematical vocabulary. ***HMH Into Math*** delivers these opportunities, which are greatly amplified through the guidance and encouragement provided for teachers, with strategies to produce growth for students within each language proficiency.

HMH Into Math also ensures equity through targeted and specific instruction. Teachers often want to group students based on the results of an assessment for differentiated nstruction or math centers. Analyzing item-level data and matching resources to address the needs of each and every student in real time ensure greater equity for greater student outcomes. Grouping students based on the valid and reliable results of an ***Into Math*** assessment is quick and easy.

HMH Into Math lessons integrate growth mindset strategies and social-emotional learning to create a culture where students and teachers embrace learning. Other features support teachers in tapping into and celebrating the unique background knowledge each student brings to the classroom.

I Can statements and scales promote students' productive perseverance and a sense of belonging as well as self-reflection.

MEETING THE NEEDS OF ALL LEARNERS

Teachers in the 21st century face both the challenge and opportunity of meeting the needs of an increasingly diverse student population, representing a wide array of cultural and linguistic backgrounds, cognitive skills, prior knowledge, readiness, interests, motivations, home situations, and learning styles. While it is critical that all students have high expectations for learning as well as access to high-quality instruction, if success is to be achieved broadly, it is also essential that all students receive the supports and differentiation each needs, regardless of their relative proficiency levels or socio-economic contexts (Aguirre et al., 2013; Gutiérrez, 2013; NCTM, 2014 & 2019; Tomlinson, 1997 & 2005). "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12).

Students struggling with mathematics benefit from early identification as well as resolutions that may prevent subsequent difficulties (Gersten, Clarke, & Mazzocco, 2007). To help all students learn to think mathematically, teachers must meet them where they are. As Vygotsky (1978) noted in his seminal research, "Optimal learning takes place within students' 'zones of proximal development'—when teachers assess students' current understanding and teach new concepts, skills, and strategies at an according level." Research continues to support the notion that for learning to be attained, activities must be at the right level for the learner (Tomlinson & Allan, 2000; Valencia, 2007) with scaffolding as called for by Vygotsky.

However, scaffolding for students who generally performing at lower levels of achievement in math cannot be provided immediately upfront before students have an opportunity to grapple with new material; if that happens, students are being denied access to rigor and then the practice is not equitable (Dixon, 2018). "All too often, so much support is provided through the initial scaffolding that the cognitive demand of the task is significantly decreased (Boston & Wilhelm, 2015). Rigor for all students is essential to equity.

In the math classroom, teachers encounter students who are on grade, above grade, below grade as well as English language learners, students with special needs, students who are gifted, and students with varying learning styles and cultural backgrounds. "[M]athematics instructors must respond to the diverse needs of individual students . . . using differentiated instruction, a process of proactively modifying instruction based on students' needs" (Chamberlin & Powers, 2010, p. 113).

Differentiated instruction is a well-established, evidence-based, organized approach to flexibly alter teaching that recognizes all learners as capable; maximizes learning for all students; and yields positive outcomes across achievement levels as well as content areas, including mathematics (Dacey et al., 2013; National Mathematics Advisory Panel, 2008; Stetson, Stetson, & Anderson, 2007; Tomlinson, 1999). Differentiated approaches to instruction recognize and support the classroom as an inclusive community where students are nourished as individual learners and provided with an appropriate, motivating balance of challenge and success. In effective differentiated environments, and all learners—those struggling and those advanced—can be successful (Lawrence-Brown, 2004). Differentiated classrooms are "responsive to students' varying readiness levels, varying interests, and varying learning profiles" (Kalbfleisch & Tomlinson, 1998, p. 54) and offer students varying levels of expectations for task completion within a lesson or unit based on their specific needs (McLeskey, Waldron, So, Swanson, & Loveland, 2001).

To differentiate instruction, teachers can adjust the content of what is being learned, adjust the process of learning (by providing additional supportive strategies, for example, or adjusting pacing), and tailor the expected outcomes (assessments, products, or tasks) of how learning is assessed (Tomlinson, 2001). In differentiation, modifications take place at the point of instruction; teachers are responsive to what happens in the classroom and are flexible in their approach to teaching, adjusting the curriculum and presentation of information to learners rather than expecting students to modify themselves for the curriculum (Strangman, et al., 2004).

Additionally, it is important to leverage the experiences students bring to the classroom; students whose backgrounds are devalued or unrecognized become alienated and disengaged from the learning process whereas when students are viewed as having lived knowledge that is a strength and resource, their learning is accelerated and their achievement is supported (Lawrence-Brown, 2004). "We argue that students need to learn mathematics in light of who they are and the diverse gifts that they bring to their experiences every day" (Aguirre et al., 2013, p. 10).

Learning is an active process of engagement and, recursively, engagement leads to motivation which leads to learning. When students are interested in what they are learning, they will spend the time and energy needed for learning to occur. Effective teachers know that students must be engaged by the content and activities presented to them to be motivated to persist in the learning process and, ultimately, to succeed in achieving learning targets (Eccles, Wigfield, & Schiefele, 1998; Guthrie & Humenick, 2004). Research shows that effective STEM education capitalizes on students' interests and experiences; identifies and build on what students know; and provides experiences to actively engage students in STEM-related practices and sustain their interest (NRC, 2011). When students are actively engaged in the process of observing, reasoning, and making connections through hands-on learning, they acquire necessary skills and ways of thinking (Stewart, Cartier & Passmore, 2005).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math provides a multitude of differentiation options that ensure growth for each and every student through resources based on individual and data-driven needs. These features support all students, and are specifically designed for multilingual learners and those from historically underserved backgrounds and those with special needs. The program offers a variety of assessment and accompanying item-level data analyses plus and teaching resources to address the progress of each and every student in real time ensure greater equity for greater student outcomes. By drilling down into the data, teachers can analyze which items students have answered incorrectly or correctly. The items can be reviewed with a class, groups, or individual students.

Review items.

Many students could use support with this standard.

The program's differentiation supports include:

- Flexible grouping aids: **Tabletop Flipchart** mini-lessons or other small-group options are provided in the **Teacher's Edition**.
- Independent Work: **On Your Own** provides independent practice to reinforce lesson content.
- Math Centers: provided collaborative centers with the leveled resources outlined in the Teacher's Edition.
- ***Waggle***: supplements ***HMH Into Math*** instruction with adaptive, targeted student practice.

HMH Into Math also offers WIDA-aligned resources specifically for English Learners. These include **Differentiated Language Routines** and **Key Vocabulary** in the Teacher Edition as well as **Point-of-Use Multilingual Differentiation** in the Teacher's Edition wrap of every lesson. The **Bilingual Math Board** includes a graphic organizer that students can use within language routines. Other supports include **Spanish Lesson Reteach** and **Multilingual Glossaries**. ***Waggle's*** robust support for English learners help students access challenging language and terms, including, idioms, cognates, and cultural references.

Teacher Tabletop Flipcharts also contain leveled scaffolded support for English learners. These scaffolding recommendations ensure teachers maintain cognitive complexity level required for mathematical reasoning while supporting students' language development.

ASSESSMENT, DATA, AND REPORTS

Assessment across a wide range of formats, timelines, and data points is fundamental to successful mathematics teaching and learning. "An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement" (NCTM, 2014, p. 5).

Historically marginalized students have also historically been disadvantaged by state and national standardized tests; high-quality assessment at the classroom level, including also diagnostic and needs-based assessment, is essential to determine how students are faring across a range of domains and what they need currently and going forward (Ed Trust, 2020; Garcìa & Weiss, 2020; Tarasawa & Samuel, 2021).

HMH Into Math provides ongoing, balanced assessment and reporting that additionally utilizes digital technologies to empower teachers with data-driven decision making and tools for effective instructional planning. *HMH Into Math* also provides grouping and resource recommendations. This solution yields critical feedback loops that encourage students' self-assessment and reflection while freeing teachers from guesswork and time-consuming assessment reporting and subsequent material selections and planning. These approaches to evaluation of learning support optimal instructional practices and drive positive outcomes for each and every student.

MONITORING STUDENT PROGRESS

Assessment is an essential component of effective instruction and a process by which teachers can continuously monitor student understanding and progress toward meeting learning goals. A wealth of studies indicates that regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning outcomes (Black & Wiliam, 1998a, 1998b, Clarke & Shinn, 2004; Kingston & Nash, 2011; Klute, Apthorp, Harlacher, & Reale, 2017; Lee, Chung, Zhang, Abedi, & Warschauer, 2020; Stecker, Fuchs, & Fuchs, 2005; Wiliam, 2010 & 2011). "[P]roviding teachers and students with specific information on how each student is performing seems to enhance achievement consistently…the effect of such practice is substantial" (Baker, Gersten, & Lee, 2002, p. 67).

"[T]eachers using assessment for learning continually look for ways in which they can generate evidence of student learning, and they use this evidence to adapt their instruction to better meet their students' learning needs" (Leahy, et al., 2005, p. 23). Effective assessment tools allow teachers to collect data about what is working and what is not so that they can take precise, swift, and effective action to better serve students. "Assessment should not merely be done *to* students; it should also be done *for* students, to guide and enhance their learning (NCTM, 2000, p. 22)

Teachers can collect a variety of variety of evidence before, during, and after instruction to evaluate progress and adjust instruction with the goal of best supporting each student. While timing of administration throughout the school year is important, it is also critical that a broad range of measures and tasks be utilized diagnostically, formatively, and summatively to compile a comprehensive picture of a student's growth and track that growth over time (National Mathematics Advisory Panel, 2008; NCTM, 2000 & 2014). Curriculum designed and developed for 21st-century learning should use formative assessment to "(a) make learning goals clear to students; (b) continuously monitor, provide feedback, and respond to students' learning progress; and (c) involve students in self- and peer assessment" (Committee on Defining Deeper Learning and 21st Century Skills, 2012, p. 182).

To make effective decisions about students' instructional needs, teachers rely on diagnostic assessment. Tailoring instruction and supplemental practice based on the results of valid diagnostic assessment improves learning outcomes (Mayes, Chase, & Walker, 2008). Diagnostic assessments provide data about students' prior knowledge and current skill levels within a domain as well as preconceptions or misunderstandings regarding learning material (Ketterlin-Geller & Yovanoff, 2009). A screening tool given to students at the opening of the school year can help identify those who are at-risk or need additional support (Fuchs & Fuchs, 2006).

Research has long established that formative assessment is also integral to an effective mathematics program (Black & Wiliam, 1998a, 1998b; Wiliam, 2018). The phrase "formative assessment" encompasses the wide variety of activities—formal and informal—that teachers employ throughout the learning process to gather this kind of instructional data to assess student understanding and to make and adapt instructional decisions. Formative assessment moves testing from the end into the middle of instruction, to guide teaching and learning as it occurs (Heritage, 2007; Tibbitt, 2020). According to Wiliam's 2010 (framework for formative assessment in math learning embeds the following key processes within instruction: making goals, making progress toward the goals, and making better progress.

In its review of studies examining formative assessment, the National Mathematics Advisory Panel (2008) concluded that "use of formative assessments benefited students at all ability levels" (p. 46). However, formative assessment is especially beneficial for lower-performing and at-risk students, including those historically underserved due to ethnicity, poverty, and disabilities and those enrolled in special education programs; monitoring student progress and directly involving students in the classroom assessment process shrinks achievement gaps and improve overall achievement (Black & Wiliam, 1998a & 1998b; NCTM, 2020; Tibbitt, 2020; Xenofontoas, 2019).

In a study of curriculum-based measurement, when teachers administered outcomes-based assessments regularly to monitor student progress and used data to make appropriate adjustments to instruction, students showed significant gains (Stecker, Fuchs, & Fuchs, 2005). Research also shows that regularly assessing and providing feedback to students on their formative assessment is a highly effective tool for teachers to produce significant—and often substantial—gains in student learning and performance (Black & Wiliam, 1998a & 1998b).

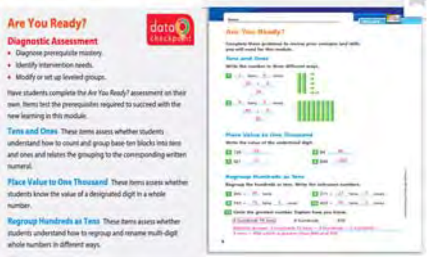
The benefits of effective classroom assessment practices are augmented when students are given ongoing opportunities for self-initiated metacognitive self-reflection (Desoete & De Craene, 2019; Lee et al., 2020; Schneider & Artelt, 2010; Wiliam, 2010). In a study of the impact of metacognitive assessment on mathematics achievement, it was found that, on both a posttest and a retention test, students who practiced reflective strategies performed significantly higher than students who did not use the strategies (Bond & Ellis, 2013).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math provides variety of options for ongoing assessment and aids to monitor of student progress and flexibly adjust instruction based on data about class and individual needs. These resources are available at the module and lesson level. Format and features include the following:

- A **Prerequisite Skills Inventory** is given at the beginning of each module.
- **Formative assessments** are available for every lesson.
- Learning aids are available—including helpful hints, corrective feedback, and multiple attempts.
- **Wrap Ups** to summarize learning and check understanding via **Exit Tickets** (short tasks or problems that indicates mastery of the lesson); **Put It in Writing** (journal prompts that require students to explain their understanding; and I Can Scales (a scale from 1 to 4 to promote student self-reflection).
- Grouping recommendations are provided from **Are You Ready?** test results.
- Frequent flexible grouping occurs 10–20 times per year compared to typical 2–3 times per year.
- Digital lessons and assessments are available by teacher assignment.
- Additional downloadable resources are available for the class and student level.
- **Interactive reports** are available by skill/standards.
- Two forms of each **Module Test** are available with multiple assessment item types.
- Constructed-response items are available in the print version.
- Unit-level performance tasks are available in the print version.
- Print and online versions are available.
- Spanish audio and text are available at Grades K–2.
- Spanish text is available at Grades 3–12.

Formal and informal assessments throughout the learning arc design and lesson setup create opportunities to check if students are mastering content and help teachers when differentiating instruction based on student performance and data.



Are You Ready?

Diagnostic Assessment

- Diagnose prerequisite mastery.
- Identify intervention needs.
- Modify or set up leveled groups.

Have students complete the *Are You Ready?* assessment on their own. Items test the prerequisites required to succeed with the new learning in this module.

Tens and Ones These items assess whether students understand how to count and group base-ten blocks into tens and ones and relates the grouping to the corresponding written numeral.

Place Value to One Thousand These items assess whether students know the value of a designated digit in a whole number.

Regroup Hundreds as Tens These items assess whether students understand how to regroup and rename multi-digit whole numbers in different ways.

Ed®, the HMH learning platform, makes it possible for teachers to effectively and efficiently leverage data for instruction. Interactive scoring reports show progress, identify needs, and lead to targeted and specific support for students at any level. After administering any assessment, teachers can immediately review class performance on *Ed* and quickly see a class-level breakdown of performance, as well as which item(s) should be reviewed with which students.

From there, teachers can select "Groups" to have *Ed* sort the students into performance groups automatically. Once students are sorted, teachers can adjust any groups by moving students around and naming each group. Teachers can also then search for relevant resources by standards and the curriculum table of contents. Small groups are one way of differentiating to address targeted and specific needs of the groups.

EVALUATING STUDENT ACHIEVEMENT

"The results of large-scale mathematics assessments should not be used as the sole source of information to make high-stakes decisions about schools, teachers, and students. High-stakes decisions should also take into account relevant and valid data on classroom-based performance, such as formative and summative assessments of high quality that offer students a range of opportunities to demonstrate their mathematical knowledge. Moreover, educational systems— states, districts, and schools—should be held accountable for providing essential support for high-quality mathematics teaching and learning before teachers and students are held accountable for assessment results" (NCTM, 2016, online).

Summative assessment differs from those that that are formative or diagnostic nature because the purpose of summative assessment is to determine the student's overall achievement in a specific area of learning at a particular time (Harlen, 2005; Moss, 2013). While traditionally summative assessment has been associated with higher stakes testing, there is a role for summative assessment in the classroom when used as an additional constructive measure demonstrating progress at a particular point of time. Evaluating student learning at the end of a unit or chapter provides insight when used as a point of information to guide subsequent instruction (Black & Wiliam, 1998a & 1998b).

Teachers can effectively use summative assessments as another measure, another point in time, and another means by which to best evaluate student understanding. As part of an integrated assessment system, summative measures can also help teachers shape instruction and differentiate to personalize learning. Summative assessments are also useful as accountability measures for grading and gauging student learning against a set of standards or expectations. Summative assessments provide evaluative information to teachers about the effectiveness of their instructional program. Research indicates that classroom summative assessments also have the potential to positively impact learning (harlen, 2005; moss, 2013; nctm, 2016).

Performance-based measures are also an important component within an effective mathematics assessment system. Performance assessments connect to the important content and process skills emphasized in instruction and offer the opportunity for students to show how well they can use what they know to classify, compare, analyze, or evaluate (Hibbard, 1996), solve problems, and create a response or product. Performance-based tasks may take different forms, require different types of performances, and be used for different purposes (formative or summative), but they are typically couched in an authentic or real-life scenario and require high-level thinking. Performance-based assessments look like what we want students to do in the classroom (Fox, 2004)—and, as a result, can inform classroom practice in positive ways. Performance tasks allow teachers to engage students in real-world activities; they "emulate the context or conditions in which the intended knowledge or skills are actually applied" (American Educational Research Association [AERA], American Psychological Association [APA], and National Council on Measurement in Education [NCME], 1999, p. 137). They model "what is important to teach and … what is important to learn" (Lane, 2013, p. 313).

According to Zimmerman, Maker & Alfaiz (2020), performance-based assessments in science, technology, engineering, and mathematics (STEM) provide an alternative and complement to standardized achievement tests because they enable a holistic evaluation of the performance of an individual student. Additionally, performance-based assessments have the potential to identify and support exceptionally talented high school students across all demographic groups, as they narrow disparities in scores among diverse cultural and economic groups and allow students to demonstrate their understanding of scientific principles and their ability to develop solutions during hands-on activities. For these reasons of equity and accurate representation of individual students' knowledge and skills, performance-based assessment appropriate for students from low SES levels are essential.

Assessment systems in high-performing nations "emphasize deep knowledge of core concepts within and across the disciplines, problem solving, collaboration, analysis, synthesis, and critical thinking. As a large and increasing part of their examination systems, high-achieving nations use open-ended performance tasks …to give students opportunities to develop and demonstrate higher order thinking skills…" (Darling-Hammond, 2010, p. 3). Research has established the benefits of performance-based assessment. A review of classroom assessment practices in an age of high-stakes testing led Schneider, Egan, and Julian (2013) to conclude that "the value of high-quality performance tasks should not be diminished and should be encouraged as an important tool" (p. 66).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math provides assessment measures and tools to evaluate and track student achievement in mathematics. Current technology-enhanced item types are mirrored in digital assessments to equip students with skills for high-stakes assessments. Teachers can also customize all module and lesson assessments in both English and Spanish. Practice for each mathematics standard is also included in the assessment system.

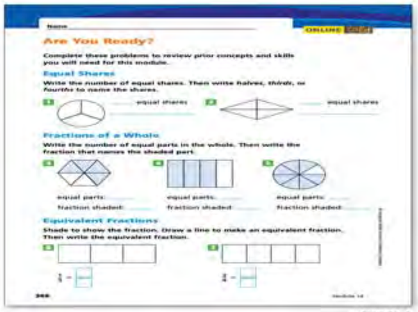
The *HMH Into Math* assessment system includes continuous growth monitoring. Growth Reports are available digitally on *Ed* and provide detailed analysis of student performance. The Student Growth Measure is an adaptive math assessment that measures growth and longitudinal progress. Features of the student growth measure include the following:

- Assessments are administered three times a year to efficiently pinpoint student proficiency in ~40 minutes.
- Adaptive assessments adjust to each student's proficiency on the fly. No two assessments are identical; items will be closely matched to each student's ability.
- Student scores are immediately available in Quantile measures, which enables monitoring of each student's growth within and across school years against grade-level proficiency.
- Actionable reports visualize student trajectory toward grade-level expectations and eventually toward proficiency on the state assessments.
- Spanish audio available at Grades K–2.

Within a school year and across school years, growth monitoring ensures that students have the skills to meet state standards and advance to higher-level mathematical thinking. Data is displayed in a variety of representations and users can drill down into data for thorough insights into performance.

The assessment options in *HMH Into Math* allow teachers to monitor, predict, and accelerate growth in students as they develop mathematics proficiency. With digital and print working seamlessly together, in-depth, embedded assessment is presented with ease and accountability. With the program's growth measures, educators can track individual students' progress in meeting specific standards and benchmarks, monitoring, tracing, and reporting mathematical proficiency over time as it develops.

Both print and digital assessment options are available to meet schools' needs. Print versions of the digital assessments are available in the Assessment Guide and on *Ed*. All digital assessments offer auto-scoring, immediate access to student data, reports, and standards correlations. Online item types include traditional multiple choice as well as technology-enhanced item types similar to what students will encounter on high-stakes assessments.



Grade 4

SUPPORTING DATA-DRIVEN DECISION MAKING

Using diagnostic data to inform solutions is essential to responsive, equitable approaches to teaching. As NCTM (2020) calls for, ongoing effective formative assessment must undergird and support learning interventions for individual students. A wealth of studies indicates that regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning (Clarke & Shinn, 2004; Fuchs, 2004, Lembke & Foegan, 2005). "One specific finding is that providing teachers and students with specific information on how each student is performing seems to enhance achievement consistently…the effect of such practice is substantial" (Baker, et al., 2002, p. 67).

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions (NCTM, 2014). By addressing the goals within mathematics learning progressions, teachers have the opportunity to examine and monitor student progress and needs in order to adjust instruction as necessary (Sztajn, et al., 2012).

Students' progress on grade-appropriate tasks must be continually monitored so that interventions can be adjusted according to students' evolving needs (Czupryk, 2020; Steiner & Weissberg, 2020). "[T]eachers using assessment for learning continually look for ways in which they can generate evidence of student learning, and they use this evidence to adapt their instruction to better meet their students' learning needs" (Leahy et al., 2005, p. 23). Effective assessment tools allow teachers to collect data about what is working—and what is not so that they can take precise, swift, and effective action to better serve students. "Assessment should not merely be done *to* students; it should also be done *for* students, to guide and enhance their learning (NCTM, 2000, p. 22)

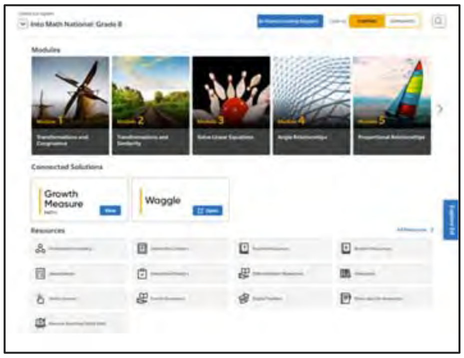
Data-driven instructional decision making is the systematic collection, analysis, and application of many forms of data from multiple sources in order to identify students' strengths and weaknesses regarding learning objectives and subsequently address student learning needs and optimize performance in future instruction. Rigorous, ongoing formative assessment that yields meaningful data is a fundamental component with an effective data-driven decision-making system (Bambrick-Santoyo, 2014; Dunn, Airola, Lo, & Garrison, 2013; Marsh, Pane, & Hamilton, 2006).

Research indicates that, when well-implemented, data-driven instruction has the potential to dramatically improve student achievement (Bambrick-Santoyo, 2014; Schifter, Natarajan, Ketelhut, & Kirchgessner, 2014).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math integrates assessment with instruction and practice. ***HMH Into Math***, ***Waggle***, and ***Growth Measure*** connect on ***Ed***, the HMH learning platform that offers SSO and accessibility from anywhere with an internet connection. The connected programs work together to provide students and teachers with best-in-class core programming, personalized supplemental practice and instruction, and a reliable benchmark assessment. Results from ***Growth Measure*** directly feed into ***Waggle***, HMH's adaptive supplemental solution, to place students in relevant practice based on domain and grade-level readiness.

Teachers receive alerts on the ***Waggle*** dashboard to facilitate instruction with skills-based differentiation in ***HMH Into Math's*** program structure and virtual and blended learning environments. This interconnected approach to assessment, core curricula, assessment, supplemental support, and curated professional learning leverages assessment data to provide timely intervention and differentiation and optimizes teacher efforts.



HMH Into Math empowers teachers with actionable insights. Whether teachers need to differentiate at the individual or small-group level, ***HMH Into Math*** with ***Waggle*** makes the right tools readily available. ***Waggle's*** actionable data insights pinpoint precise skill gaps in real time, assessing students' knowledge without requiring a diagnostic or summative test.

HMH Into Math provides valuable and reliable data every step of the way. The teacher is enabled to focus time on how to differentiate to meet each student's need. Ongoing, varied assessments contributes to the whole mathematical learning picture of each student and is critical in uncovering how to best support every student's growth.

Class-level scores on program assessments (module/unit and lesson-level assessments) and overall class proficiency can be viewed.

Ed reports are interactive. Selecting a part of the report will reveal a drill-down into the data.

Single assessment reports show student proficiency by standard as well as item-level analysis and usage data..

DIGITAL LEARNING EXPERIENCE

Over the past decade, policies and practices regarding technology use in classrooms around the country have shifted incrementally to widespread—and widely varying—application. Concurrent with such trends, there has been an emergence of growing evidence attesting to the positive impacts of technology in education as well as profound advances and innovations within the technology itself. No longer a question of whether technology can improve learning, the issues became *how* to enable technology to deliver improved learning outcomes for all students. Since the start of the 21st century, educators in United States have broadly adopted the understanding that "[t]echnology can be a powerful tool for transforming learning. It can help affirm and advance relationships between educators and students, reinvent our approaches to learning and collaboration, shrink long-standing equity and accessibility gaps, and adapt learning experiences to meet the needs of all learners" (U.S. Department of Education, 2018, p. 3).

But when the global pandemic hit in 2020, digital learning suddenly, profoundly became—rather than a means of improving education—a critical mission, the only way of providing instruction to students remotely. As Fisher, Fry, and Hattie (2020) noted, teaching in 2020 wasn't so much distance learning as crisis teaching. While the impacts of COVID-19 will continue to present unprecedented challenges and uncertainties for schools in the years to come, one point of clarity is that the future of education will rely in some part on technology—which requires that educators have available to them resources that support effective digital and blended hybrid instruction.

HMH Into Math harnesses technology to provide interactive, adaptive, and personalized instruction along with practice and assessment solutions addressing individual students' ongoing needs.

BEST PRACTICES IN DIGITAL MATHEMATICS LEARNING

The U.S. Department of Education (2019) stresses how technology plays a central role in STEM education in terms of both its role within the STEM professions today's students are being trained for as well as the potential technology has to significantly improve both experiences and outcomes for students as they learn STEM concepts and build STEM knowledge throughout their K-12 educations. Per the USDOE, technology can effectively be leveraged to support the following critical components of effective mathematics instruction:

- Dynamic representations: Students can more effectively develop STEM concepts via interactions with digital models, simulations, and dynamic representations of mathematical, scientific, and engineering systems.
- Collaborative reasoning: Technology platforms support students' collaborative discussion and shared construction of STEM concepts, fostering engagement and equalizing participation among group members, as well as yielding higher performance on test measures.
- Immediate and individualized feedback: Digital tools provide students with prompt and customized feedback as they practice or demonstrate their STEM skills that yield faster and improved learning outcomes.
- Computational thinking: students can use technology to engage in formulation, analysis, and solving of problems using algorithms, data, and simulations to investigate questions and build new understandings about phenomena.
- Project-based interdisciplinary learning: both process and product are enriched when students utilize technology tools in the context of authentic project- or challenge-based learning activities that integrate multiple STEM fields. Technology can also be used effectively to support task management.
- Embedded assessments: assessments aligned to ongoing STEM instruction and delivered digitally provide opportunity for students to reflect on and demonstrate and for teachers to evaluate their learning. Technology can also foster peer reviews of student work.
- Evidence-based models: students use technology to reference or create models based on data and evidence. Digital models also facilitate revisions and refinement over time, yielding improved scientific models and accompanying understanding of concepts.

Within mathematics instruction specifically, tools and technology are now indispensable. Essentials tools include manipulatives and counters and calculators, which increasingly are being digitized for classroom use and integrated with digital tools such as tablets and whiteboards, offered within fully or partly digital curricular content. The technology is continually evolving. But at a most basic level, what's most critical is the functionality of the digital tools and platform. "An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (NCTM, 2014, p. 78).

Before COVID-19 drove educators around the United States and the world to suddenly switch to remote teaching in early 2020, the number of students receiving instruction in online and blended learning environments had been steadily growing (Gemin & Pape, 2017; Graham, Borup, Pulham, & Larsen, 2019). While the field of research is relatively new, findings that emerged over the past two decades indicate that digital learning has enormous potential to positively transform education for diverse groups of students (Abdoolatiff & Narod, 2009; Patrick & Powell, 2009; USDOE, 2016 & 2010). Improvements in student-centered, cooperative, and higher order learning as well as problem solving and writing skills have been found within computer-intensive classroom settings (Ross, Morrison & Lowther, 2010). In 2016, U.S. Department of Education reported that technology-intensive instruction can make education more equitable by closing the digital use divide and making transformative learning opportunities available to all students.

Blended learning utilizes both device-driven, technology-based instruction and face-to-face instruction in a conventional classroom context, with the objective to maximize the advantages of each. Research findings on the effects of blended learning are strikingly positive (Delgado, Wardlow, McKnight & O'Malley, 2015; Graham, Borup, Pulham, & Larsen, 2019; Osguthorpe & Graham, 2003; Tamim, Bernard, Borokhovski, Abrami & Schmid, 2011). In a meta-analysis examining online and traditional face-to-face instruction with mixes of both, blended instruction emerged as the most effective of the three approaches (USDOE, 2010). Likely because blended learning teaches students through engaging media and modes that fit with their daily practices and experiences, students tend to view blended learning favorably (Uğur, Akkoyunlu, & Kurbanoğlu, 2011). Blended learning opportunities specifically expand the possibility of growth for all students while affording historically disadvantaged students' greater equity of access to high-quality education, in the form of both enhanced, instructionally effective content and more personalized learning (Molnar, 2014). "[B]lended learning that combines digital instruction with live, accountable teachers holds unique promise to improve student outcomes dramatically" (Public Impact, 2013, p. 1).

A well established body of evidence supports the position that effective technology use in the classroom, through web-based and multimedia learning, increases student engagement and motivation (Abdoolatiff & Narod, 2009; Chen, Lambert, & Guidry, 2010; Mayer, 2013; Reinking, 2001; Taylor & Parsons, 2011, Tucker, 2012). Game-based learning as well as simulations and virtual learning experiences have also widely shown positive effects on learning (Henderson, Klemes, & Eshet, 2000; Merchant, Goetz, Cifuentes, Keeney-Kennicutt, Kwok & Davis, 2012).

However, research also suggests that the best practices in blended learning reflect the same from those of traditional classrooms, but with some critical adaptations within the digital environment (Borup & Archambault, 2018). To achieve optimal growth, blended learning should support teachers in being flexible and responsive to students, to integrate multiple data sources into their constant stream of formative assessment, and to deliberately incorporate more rigorous learning activities (Anthony, 2019). In a large-scale study, Kwon, Debruler, & Kennedy (2019) found that for online learning to be successful, it is important that teaching is structured so that students make steady attempts to complete learning tasks, ideally with students' own self-regulated learning scaffolded by course pacing guides.

Ultimately, it is the choice of task that matters in advancing learning—not the medium; teachers should use technology as the means and starting point, not the core of teaching (Fisher, Fry, & Hattie, 2020). As Hattie's (2018, with Clarke) ongoing findings about best practices with technology continue to affirm, instructional principles that transcend deliver format include: fostering student self-regulation to help them move toward deeper learning; increase student agency; include a diversity of instructional approaches (not just some direct instruction and then some off-line independent work); include well-designed peer learning; provide feedback within a high-trust environment integrated into the learning cycle.

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math provides a comprehensive, engaging interactive blended learning core program that is enhanced and extended by its digital features and multimedia experience. Additionally, the program accelerates growth, as *Waggle* complements the *Into Math* lesson plan, providing a digital option for online, skills-based differentiation in and out of the classroom. Students and teachers benefit from *Waggle*'s innovative language supports for students who speak multiple languages as well as its SEL framework and embedded gaming.

In addition to offering customizable support at each student's proficiency level, *Waggle* also promotes engaged in math learning with exploration of age-appropriate new worlds and missions. Students can earn badges, points, and avatar customizations as they complete assignments and demonstrate the attributes of a Growth Mindset, from seeking challenges to persevering through challenges.

HMH Growth Measure can be used to track yearly progress and provide further personalized pathways of skills-based instruction and practice. This valid and reliable student growth measure is administered digitally three times per year and designed to monitor student growth and determine grade-level expectations against Florida Statewide Assessments. All items align to Florida's B.E.S.T. Standards and detailed data reports are used guide instructional decisions and help provide individualized learning opportunities.

HMH Go™ is an app that gives students the ability to download their core digital resources for later offline use.

INCREASED AGENCY AND A MORE PERSONALIZED APPROACH TO MATHEMATICS INSTRUCTION

"Digital learning has the capacity to transform schools into new models for education that are student-centric, highly personalized for each learner, and more productive, as it delivers dramatically better results at the same or lower cost" (Horn & Staker, 2011, p. 2). Blended learning opportunities expand the possibility of growth for all students in the form of enhanced, instructionally effective and engaging content as well as more personalized learning with preferred modalities; agency over the pace of their own learning; and more frequent and timely feedback—while affording historically disadvantaged students additional benefits via greater equity of access to high-quality education (Horn & Staker, 2011; Imbriale, 2013; Molnar, 2014; Public Impact, 2013; Tucker, 2012; USDOE, 2016).

Digital programming offers an additional benefit of increased automation, which can significantly simplify educators' lives by eliminating low-value manual tasks such as attendance records and student assessment data entry) as well as free up educators' so that more of their time and energies can be dedicated to small group, 1:1, or other effective deliveries of direct instruction. The further impact of allowing the platform to capture student achievement data in real time is a freeing up of resources so that educators can "take advantage of the things that leading brick-and-mortar schools do well, such as creating a strong, supportive culture that promotes rigor and high expectations for all students, as well as providing healthy, supportive relationships and mentorship." (Horn & Staker, 2011, p. 7)

Other researchers have indicated that multimedia learning leads to increased student motivation because of the responsiveness and student control these environments allow and the subsequent engagement in active learning (Schunk, Pintrich, & Meece, 2008; Sims, Dobbs & Hand, 2002). Zhang (2005) found students in a full interactive multimedia-based e-learning environment achieved better performance and higher levels of satisfaction than those in a traditional classroom and those in a less interactive e-learning environment, with a lack of control over content diminishing potential benefits. "This study implies that to create effective learning, e-learning environments should provide interactive instructional content that learners can view on a personalized self-directed basis" (p. 160).

A blended learning approach specifically offers a more consistent and personalized pedagogy helps each child feel and be successful at school (Kwon et al., 2019). Digital learning tools can provide more flexibility and support for individual students by modifying content and complexity; additionally, advances in software technology have increased adaptive learning and improved feedback. By providing a diverse array of online and other digital resources, technology supports learning drawn from real-world challenges and students' personal interests and passions while also aiding the organization of a project-based curriculum (USDOE, 2016).

Digital learning can also increase the capacity for students to work together. Computer-based collaborative tools allow for online interactions that can create and strengthen a community of learners while fostering students' communication and collaboration skills (Tucker, 2012). "What makes blended learning particularly effective is its ability to facilitate a community of inquiry" (Garrison & Kanuka, 2004, p. 97).

Research shows that effective technology use in the classroom motivates students to take charge of their own learning and that digital learning itself is enhanced when students are given more control over their interaction with media (Horn & Staker, 2011; Patrick & Powell, 2009; USDOE, 2010). Technology is increasingly being utilized in the United States to personalize learning and give students more choice over what and how they learn, and at what pace; this will better prepare students to organize and direct their learning in their lives even after formal schooling (USDOE, 2016). "Online learning has the potential to transform teaching and learning by redesigning traditional classroom instructional approaches, personalizing instruction, and enhancing the quality of learning experiences. The preliminary research shows promise for online learning as an effective alternative for improving student performance across diverse groups of students (Patrick & Powell, 2009, p. 9).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH Into Math in conjunction with ***Waggle*** goes beyond adaptive learning to truly personalize practice and instruction to support students at all proficiency levels. Keeping students engaged in their Zones of Proximal Development is key to creating growth. Waggle uses 13 different data points to personalize learning for every single student.

A robust digital platform makes assessment and monitoring reports customizable and accessible for teachers and students.

HMH Into Math reports support growth and offer the following features:

- Real-time data, reports, and analytics will be provided for digital assessments, investing in the teacher to increase his/her efficiency to focus on the students' needs based on the needs identified.
- Many reports will be available for teachers to monitor student outcomes and intervene or challenge students more effectively.
- Reports will be provided at the student, class, and school levels. Some common reports are highlighted.

.

PROFESSIONAL SERVICES

To support the delivery of effective instruction, ***HMH Into Math*** features research-based approaches to professional learning that support teachers in becoming developers of high-impact learning experiences for their students. Comprehensive professional learning solutions are data and evidence driven, mapped to instructional goals, and centered on students—and they build educators' collective capacity. HMH allows teachers to achieve agency in their professional growth through effective instructional strategies, embedded teacher support, and ongoing professional learning relevant to everyday teaching.

CONTINUUM OF CONNECTED PROFESSIONAL LEARNING

Effective professional learning, whether in-person, online, or blended, takes place as a "series of connected, coordinated components on a continuum" (Rock, 2019). This continuum includes alignment between the study of theory and practice, observation of theory and practice, individual coaching, and further practice and refinement through collaboration. Each of these components is essential to support and build on the content and pedagogy that is learned, observed, and practiced in each of the other components. Long-term connected professional learning includes cohesive features—online coaching, remote peer observations, online collaboration, and facilitated online communities—all with a focus on how to ensure social and emotional well-being and meaningful student learning in digital environments. Connecting workshops to follow up learning and support among peers and with coaches can help teachers retain new knowledge, practice new skills, and share innovative effective approaches. A connection between workshops, coaching, and collaboration is essential for professional learning to make a difference in student achievement (Aguilar, 2019).

Research increasingly finds that teachers' professional learning is essential to school reform and a vital link between standards movements and student achievement (Borman & Feger, 2006; Garet et al., 2001; Gulamhussein 2013; Sweeney 2011; Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009; Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). According to Wei et al. (2009):

> As students are expected to learn more complex and analytical skills in preparation for further education and work in the 21st century, teachers must learn to teach in ways that develop higher order thinking and performance. . . . Efforts to improve student achievement can succeed only by building the capacity of teachers to improve their instructional practice and the capacity of school systems to advance teacher learning (p. 1).

Enabling educational systems to achieve on a wide scale the kind of teaching that has a substantial impact on student learning requires much more intensive and effective professional learning than has traditionally been available. If we want all young people to possess the higher-order thinking skills they need to succeed in the 21st century, we need educators who possess higher-order teaching skills and deep content knowledge. (Gov. James B. Hunt, Jr. in Wei et al.'s *Professional Learning in the Learning Profession: Status Report*, 2009, p. 2)

Current reform efforts across disciplines require significant shifts in teachers' roles from traditional, rote, fact-based approaches to fostering students' deeper engagement, critical thinking, and problem solving. For schools to support these standards and instructional practices, effective professional learning during the implementation stage, when teachers are learning and committing to an instructional approach, is critical (Gulamhussein, 2013). While technology transforms the teacher's role, this does not mean that evidence-based teaching practices should be discarded. In fact, effective instruction results when teachers purposefully combine these tools with proven instructional approaches (Kieschnick, 2017).

Teachers' initial exposure to a concept should engage them through varied approaches and active learning strategies to make sense of the new practice (Bill & Melinda Gates Foundation, 2014; Garet, Porter, Desimone, Birman, & Yoon, 2001; Gulamhussein, 2013). An effective professional learning program should focus on the targeted content, strategies, and practices (Bill & Melinda Gates Foundation; 2014; Saxe, Gearhart, & Nasir 2001; Wei, 2009) and be grounded in the teacher's grade level or discipline (Gulamhussein, 2013).

Research has documented that educational reforms are not self-implementing or predictable in terms of how they may (or may not) take hold at the classroom level; the vital link necessary for targeted change is local professional learning by teachers (Borman & Feger, 2006). Effective professional learning is embedded and ongoing as part of a wider reform effort, rather than an isolated activity or initiative (Garet et al., 2001; Wei et al., 2009). "The duration of professional development must be significant and ongoing to allow time for teachers to learn a new strategy and grapple with the implementation problem" (Gulamhussein, 2013, p. 3).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

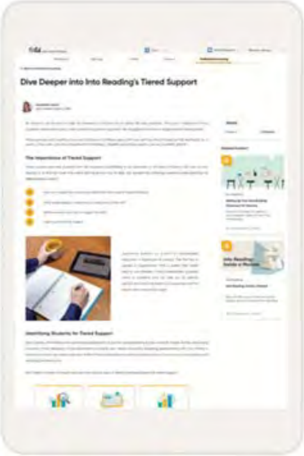
Educators have access to sustained professional development support for ***HMH Into Math.*** Your subscription includes continuous implementation support all year long. To ensure teachers are successful and confident with their new HMH program from the onset, we provide a system of support designed to concentrate on what's most important for teacher's first 30 days, which includes district-scheduled program trainings and Teacher Success Pathways on Ed, the HMH Learning Platform.

Benefits:

- Live and on-demand, solution-specific teaching resources available on their schedule
- Teachers have multiple opportunities to attend the sessions in their pathway and unlimited access to their resource materials throughout the year, no matter when in the year they are hired.
- Printable parent and caregiver letters in English and Spanish to help with at-home support and more!

What types of resources are included?

- Teacher Success Pathways are personalized to match the programs and grades they are teaching and include topics that address different elements of teaching such as planning and prioritizing instruction, assessing and differentiating, and personalizing instruction.
- Yearlong access to Teacher's Corner™ puts real-world classroom videos and best practices at your fingertips on your schedule. Plus, free Live Events allow you to build a community around solutions to today's instructional challenges.



Dive Deeper into Into Reading's Tiered Support

Ongoing professional learning and support for ***HMH Into Math*** isn't limited to teachers—Leaders can also view on-demand resources such as classroom videos and live events via *Leader's Corner.*

Leader's Corner Resources Support:

- Live Events
- Getting Started
- Program Support
- Breakroom

JOB-EMBEDDED COACHING TO STRENGTHEN TEACHING AND LEARNING

Research has demonstrated that sustained, job-embedded coaching is the most effective form of professional learning, whether it is delivered in person or in a virtual setting. Coaching delivered in person has been most effective when coaches are highly experienced and focus their work with teachers on a clearly specified instructional model or program. Other opportunities for teachers to develop their content knowledge of the targeted instructional model (e.g., in courses, workshops, or coach-led learning groups) are also an important component of successful coaching programs. Online coaching shows promise for being at least as effective as in-person coaching for improving outcomes, though the research base comparing delivery systems is thin. The balance of evidence to date, however, suggests that the medium through which coaching is delivered is less important than the quality and substance of the learning opportunities provided to teachers (Matsumura, Correnti, Walsh, DiPrima Bickel, & Zook-Howell, 2019).

A recent meta-analysis of coaching programs found effect sizes of 0.49 SD on instructional practices and 0.18 SD on student achievement (Kraft, Blazar, & Hogan, 2018). Encouragingly, teachers who received virtual coaching performed similarly to teachers who received in-person coaching for improving both instructional practices and student achievement. The authors identified several aspects of coaching in a virtual setting as potential strengths: increasing the number of teachers with whom a high-quality coach can work, reducing educators' concern about being evaluated by their coach, and lowering costs while increasing scalability (Kraft et al., 2018).

The International Society for Technology in Education (ISTE) embraces a professional development model that includes effective coaching, collaborative communities, and a technology-rich environment. Effective coaching is contextual, relevant, and ongoing. Collaborative communities can be school-based or online professional learning communities that allow teachers to learn from each other through observation, imitation, and modeling. ISTE recommends that school districts chose a coaching model that best fits the needs of their teachers, whether it is cognitive coaching, instructional coaching, or peer coaching (Beglau, Hare, Foltos, Gann, James, Jobe, Knight, & Smith, 2011). Effective professional learning programs provide continued follow-up and support from coaches (Sweeney, 2011). Knight (2011) stresses that once training initiatives are kick-started to raise awareness of targeted teaching practices, follow-up and coaching are essential: "[l]asting change does not occur without focus, support, and systemwide accountability. . . . Support is necessary for transferring talk into action" (p. 10).

Instructional coaching entailing the modeling of specific sought-after practices has been shown to help teachers embrace and implement best practices and educational policy (Coburn & Woulfin, 2012; Gulamhussein, 2013; Heineke & Polnick, 2013; Knight, 2011; Taylor & Chanter, 2016; Wei et al., 2009).

Effective modeling of targeted instructional practices is purposeful and deliberate, incorporates academic language, and is based on research (Taylor & Chanter, 2016). Gulamhussein (2013) reports that:

> While many forms of active learning help teachers decipher concepts, theories, and research-based practices in teaching, modeling—when an expert demonstrates the new practice—has been shown to be particularly successful in helping teachers understand and apply a concept and remain open to adopting it (p. 17).

According to a large-scale survey commissioned by the Bill & Melinda Gates Foundation (2014), teachers seek more opportunities to be coached in learning effective new instructional strategies and practices, believing these professional learning efforts are more valuable. "Like athletes, teachers will put newly learned skills to use—if they are coached" (Joyce & Showers, 1982, p. 5).

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

Instructional coaching, which includes lesson modeling, deepens your implementation and ensures sustainable, data-driven results.

Online and blended coaching powered by HMH Coaching Studio combines personalized professional learning with an interactive collaboration platform. Coaching provides teachers with the support they need to positively impact students every day. By powering our coaching services with Coaching Studio, a fully integrated and interactive virtual space, coaches, teachers, and other stakeholders can share resources, engage in meaningful discussions, and reflect on their learning at any time, in one centralized location. Through the HMH Coaching Studio platform, teachers set and track progress on their learning goals, stay connected to their HMH coach and peers between live and online coaching sessions, upload their own resources, and have access to a library of on-demand lesson-modeling videos and resources for "anytime" learning. The HMH Coaching Studio can be accessed via computer and mobile.

The Coaching Studio supports continuing collaboration and on-demand learning. Teachers have access to a private coaching space as well as a library of on-demand videos that model lessons. The platform makes it easy for teachers and coaches to stay connected, share resources, and upload and reflect on classroom videos, and make continuing progress on learning goals. Coaching Studio aligns to the HMH Coaching Model. Teachers can easily share artifacts with their coach and peers via the mobile app or the website

An HMH coach and teacher(s) will

- Analyze student work to Set student learning targets and next steps.
- Learn New Instructional Skills - Access best practice classroom videos and resources aligned to the goals they have set, discussing these skills with their HMH coach and peers.
- Apply Learning in the Classroom - Collaboration with an HMH Coach is easy- teacher(s) and coaches can upload videos, images, and documents aligned to their work with students.
- Review Progress & Reflect on Results - Video-based self or collaborative reflection. Teachers and coaches can share feedback, make notes, tag key strategies, connect thoughts to helpful resources, and align artifacts to HMH Implementation Frameworks.

Dive Deeper into Into Reading's Tiered Support

The Importance of Tiered Support

Identifying Students for Tiered Support

PERSONALIZED & ACTIONABLE PROFESSIONAL LEARNING

Personalized professional development allows teachers to pursue learning to support their instructional needs in their own place and at their own pace. Teachers can take courses via online professional learning portals, opportunities offered by the school, or off-campus settings. In this process, teachers learn new competencies, demonstrate what they have learned in their classrooms, and submit evidence of mastery. As teachers build their knowledge and skills, they earn badges to demonstrate their expertise (Clayton, Elliott, & Iwata, 2014).

Many school districts and providers of teachers' professional development are moving toward a more personalized model of professional development, taking a cue from the movement toward personalized learning for students. This approach often focuses on short modules, which teachers can choose and then complete on their own time. The modules can incorporate aspects of gamification, micro-credentialing, and online professional development communities. By allowing teachers to choose their own professional development courses and activities, the professional development will be better matched to their needs. Teachers will be able to set goals, find resources to help them meet those goals, track their progress, and get feedback from supervisors and colleagues (Gamrat, Zimmerman, Dudek, & Peck, 2014; Meeuwse & Mason, 2017).

Effective training efforts should be developed according to evidence-based strategies for adult learning and communication, including engaging teachers in varied approaches that allow for their active participation (Bill & Melinda Gates Foundation, 2014; Garet et al., 2001; Gulamhussein, 2013; Guskey, 2002; Taylor & Chanter, 2016). As intellectuals, they are empowered to reflect on theory, research, and their practice to innovate and implement new teaching strategies and approaches. This process of reflection can lead to teachers' turning to their colleagues for advice and clarification—a process sometimes called "collective sensemaking," which research has shown that in the form of professional learning communities can be a powerful motivator for school improvement (Coburn, 2005).

As Bryk and colleagues (2015) noted in a study of improvement efforts that included professional learning, positive changes happen in the presence of teachers' "good will and engagement," which is often rooted in teachers having choice and autonomy in their own learning. These qualities are essential whether teachers meet for large-group professional learning, attend professional learning communities within their schools, or work on their own to search out experts to guide them through self-study with print or online resources.

Teachers who seek to improve their practice and their students' achievement can also turn to resources to help them continue successfully on their path toward professional mastery and control the place, pace, and path of their professional learning. Individually and collaboratively, they engage in a process sometimes called "self-coaching" (Wood et al., 2014). There are five steps to self-coaching that align with high-quality teaching:

1. Collecting data to help answer questions about instructional improvement. Formative and benchmark data are important, but so is information about students' interests, styles of learning, and work habits.
2. Reflecting on the data as a whole and on the data that results from looking back on each day's instruction, and each week's instruction.
3. Acting on the reflections, trying things out, and sharing the results of teachers' actions in a collaborative and mutually supportive group.
4. Evaluating one's practice, especially through video self-reflection, asking questions about effectiveness of instruction and students' receptivity to the instruction.
5. Extending one's actions, for example, taking a successful approach to teaching students to understand complex narrative texts to instruction on reading, social studies, science, or other informational texts.

HOW *HMH INTO MATH* ALIGNS TO RESEARCH

HMH offers online and on-demand resources that support and enhance the teaching and learning journey. As a partner (Professional Learning Partner Guide), HMH has demonstrated that we understand students' diverse needs and have the right professional learning tools to support educators' ever-changing needs. We deliver ongoing professional learning on topics that matter to you and your students.

HMH's Live Online Course modules allow you to design flexible and ongoing professional learning courses aligned to your instructional initiatives. A Live Online Course consists of six, 1-hour modules for up to 35 participants that can be delivered over time and an additional one-hour consultative planning session.

Courses provide both course-specific and agnostic topics, including:

- SEL
- Cultural Responsiveness
- Remote teaching and learning
- Teaching Children to read
- Math Discourse
- And many more…

APPENDIX

A NOTE ON THE COVID-19 PANDEMIC

Educators in the 21st century face daunting challenges. Such challenges were exacerbated in 2020-21 as COVID-19 necessitated near universal school closures and instruction at every level was disrupted. Further, the global pandemic exposed and amplified pervasive inequities impacting historically marginalized groups, including children from low-income backgrounds, children of color, multilingual learners, and children with disabilities. The nation's most vulnerable students—those suffering within education policies and infrastructures that systemically underserve them have been—were disproportionately adversely affected by the academic, economic, and public health tolls COVID-19 (Ed Trust, 2020; Garcia & Weiss, 2020; Terada, 2020). As schools resume in-person instruction and remedy the interrupted learning, the importance of meeting the needs of all students has perhaps never been more urgent.

Guidance on how to best serve students through continued uncertain times is available in established and new research. Mathematics is an area of particular concern. Initial examinations into the of COVID-19 on K-12 education show significant learning loss in mathematics, especially for disadvantage students (Kuhfeld, Tarasawa, Johnson, Ruzek & Lewis, 2020). This was not surprising given that math learning successively builds on conceptual understandings and foundational skills—but this mean that math must be an area of prioritized focus, with efforts aimed at making up for grounds and gains lost and identifying and prioritizing what is most vitally taught at specific grade levels (Council of the Great City Schools, 2020). It will be critical that districts expand investments that support effective mathematics instruction in order to help students recover from widespread disruptions. Specifically, districts must consider the extent to which organizational structures establish and maintain high expectations for all students, while allowing for the customization of learning concepts based on data (Tarasawa & Samuel, 2021).

A wide range of approaches to curricula and supplemental programming exist to boost academic achievement. There is no one-size-fits-all solution and, on macro- and micro-levels schools continually have to make based on the needs of students and communities they serve as well as the resources they have available—and the short- and long-term impacts of COVID-19 related school closures will undoubtedly such challenges all the more daunting. Specifically, districts must consider the extent to which organizational structures establish and maintain high expectations for all students, while allowing for the customization of learning concepts based on data (Tarasawa & Samuel, 2021).

WORKS CITED

Abassian, A., Safi, F., Bush, S., & Bostic, J. (2020). Five different perspectives on mathematical modeling in mathematics education. *Investigations in Mathematics Learning*, *12*(1), 53-65.

Abdoolatiff, S., & Narod, F. B. (2009). Investigating the effectiveness of computer simulations in the teaching of "atomic structure and bonding." *Chemistry Education in the ICT Age,* 85-100.

Achieve. (2010). Comparing the Common Core State Standards in Mathematics and NCTM's Curriculum Focal Points. Washington, DC: Author.

Aguilar, E. (2019). You can't have a coaching culture without a structure. *Educational Leadership: A Culture of Coaching*, 77(3), 22–28.

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). Partnering with Families and Communities to Support Children's Equitable Mathematics Learning. In *The Impact of Identity in K-8 Mathematics Learning and Teaching: Rethinking Equity-Based Practices*. The National Council of Teachers of Mathematics, Inc.

Alberti, S. (2012). Making the shifts. *Educational leadership*, *70*(4), 24-27.

Almarode, J. & Vardas, K. (2018). *Clarity for Learning: Five Essential Practices that Empower Students and Teachers*. Thousand Oaks, CA: Corwin-Sage Publications.

American Educational Research Association [AERA], American Psychological Association [APA], & National Council on Measurement in Education [NCME]. (1999). *Standards for educational and psychological testing.* Washington, DC: American Educational Research Association.

Anthony, E. (2019). (Blended) Learning: How traditional best teaching practices impact blended elementary classrooms. *Journal of Online Learning Research*, 5(1), 25-48.

Anwar, S., Bascou, N. A., Menekse, M., & Kardgar, A. (2019). A systematic review of studies on educational robotics. *Journal of Pre-College Engineering Education Research (J-PEER)*, *9*(2), 2.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52(3), 215–241.

Asturias Méndez, L. H. (2015). Access for All: Linking Learning and Language. English Learner Leadership Conference presentation, Sacramento County Office of Education. Retrieved online: https://www.scoe.org/files/el15-asturias.pdf

Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, 103(1), 51-73.

Bambrick-Santoyo P. (2014). Make Students College-Ready in High School. *Phi Delta Kappan*, 95(5), 72-73.

Baroody, A.J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, 13(1), 22–32.

Baroody, A. J., Bajwa, N. P., & Eiland, M. (2009). Why can't Johnny remember the basic facts?. *Developmental disabilities research reviews*, *15*(1), 69-79.

Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: an alternative form of communication for academically low-achieving students. *Learning Disabilities Research & Practice*, *20*(2), 119-135.

Beane, J. A (1997). *Curriculum integration: Designing the core of democratic education*. Alexandria, VA: Association for Supervision and Curriculum Development.

Beglau, M., Hare, J. C., Foltos, L, Gann, K., James, J., Jobe, H., Knight, J., & Smith, B. (2011). *Technology, coaching, and community: Power partners for improved professional development in primary and secondary education* (White Paper). International Society for Technology in Education (ISTE). https://www.ri-iste.org/Resources/Documents/Coaching_Whitepaper_digital.pdf

Berry, W. (2018). Thinking about instructional routines in teaching and learning mathematics. NCTM Messages from the President. Retrieved online: https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Robert-Q_-Berry-III/Thinking-about-Instructional-Routines-in-Mathematics-Teaching-and-Learning

Bill & Melinda Gates Foundation. (2014). Teachers know best: Teachers' views on professional development. Seattle, WA: Author.

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003, April). A successful intervention—Why did it work. In *American Educational Research Association annual meeting, Chicago*.

Black, P., & Wiliam, D. (1998a). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan*, 80(2), 139–148.

Black, P., & Wiliam, D. (1998b). Assessment and classroom learning. *Assessment in Education: Principles, Policy, and Practice*, 5(1), 7–73.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child development*, *78*(1), 246-263.

Blatto-Vallee, G. Kelly, R. Gaustad, M. Porter, J. & Fonzi, J. (2007). Visual spatial representation in mathematical problem solving by deaf and hearing students. *Journal of Deaf Students and Deaf Education*, 12(4), 432-48.

Bobis, J. & Way, J. (2018). Building connections between children's representations and their conceptual development in mathematics. In Kinnear V., Lai M., Muir T. (Eds) *Forging Connections in Early Mathematics Teaching and Learning*. Early Mathematics Learning and Development. Springer, Singapore.

Bond, J. B., & Ellis, A. K. (2013). The effects of metacognitive reflective assessment on fifth and sixth graders' mathematics achievement. School Science and Mathematics, 113(5), 227–234.

Borman, J., & S. Feger. (2006). *Instructional coaching: Key themes from the literature*. Providence, RI: The Education Alliance at Brown University.

Borup, J., & Archambault, L. (2018). K-12 blended and online competencies, standards, retention, and attitudes. *Journal of Online Learning Research*, *4*(1), 1-3.

Boston, M. D., & Wilhelm, A. G. (2015). Middle school mathematics instruction in instructionally focused urban districts. *Urban Education*, *52*(7), 829-861.

Bouck, E. C., & Park, J. (2018). A systematic review of the literature on mathematics manipulatives to support students with disabilities. *Education and Treatment of Children*, *41*(1), 65-106.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (1999). *How people learn: Brain, mind, experience, and school*. National Research Council. Washington, DC: National Academies Press.

Bray, W. S. (2013). How to leverage the potential of mathematical errors. *Teaching children mathematics*, *19*(7), 424-431.

Bray, W.S., Dixon, J.K., & Martinez, M. (2006). Fostering communication about measuring area in a transitional language class. *Teaching Children Mathematics*, 13(3), 132–138.

Bryk, A., Gomez, L. M., Grunnow, A., & LeMahieu, P. G. (2015). *Learning to improve: How America's schools can get better at getting better.* Cambridge, MA: Harvard Education Press.

Burns, M. (2004). Writing in math. *Educational Leadership*, *62*(2), 30-33.

Caine, R. N., & Caine, G. (1991). *Making connections: Teaching and the human brain*. Alexandria, Virginia: Association for Supervision and Curriculum Development.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*(2), 380–400.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (2015). *Children's mathematics. Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T.P., Franke, M.L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary schools*. Portsmouth, NH: Heinemann.

Chamberlin, M. T. (2005). Teachers' discussions of students' thinking: Meeting the challenge of attending to students' thinking. *Journal of Mathematics Teacher Education* 8(2), 141–70.

Chamberlin, M.T., & Powers, R.A. (2010). The promise of differentiated instruction for enhancing the mathematical understandings of college students. *Teaching Mathematics and its Applications: An International Journal of the Institute of Mathematics and its Applications*, 29(3), 113–139.

Chapin, S.H., O'Connor, C., & Canavan Anderson, N. (2003). *Classroom discussions: Using Math Talk to help students learn, Grades 1-6*. Sausalito, CA: Math Solutions Publication.

Charles, R.I. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership*, 7(3), 9–24.

Chen, P., A. Lambert & K. Guidry. (2009). Engaging online learners: The impact of Web-based learning engagement on college student engagement. *Computers & Education*, 54, 1222-1232.

Childs, K. J., & Glenn-White, V. (2018). Posing purposeful questions through making sense of mathematical tasks. *SRATE Journal*, *27*(2), 11-17.

Clarke, B., & Shinn, M.R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review*, 33(2), 234-248.

Clarke, S., Timperley, H., & Hattie, J. (2004). *Unlocking Formative Assessment: Practical Strategies for Enhancing Students' Learning in the Primary and Intermediate Classroom*. Auckland, New Zealand: Hodder Moa Beckett.

Clayton, J., Elliott, R., & Iwata, J. (2014). Exploring the use of micro-credentialing and digital badges in learning environments to encourage motivation to learn and achieve. In B. Hegarty, J. McDonald, & S.K. Loke (Eds.), *Rhetoric and reality: Critical perspectives on educational technology.* Proceedings ascilite Dunedin 2014 (pp. 703–707).

Clements, D. H. (2000). 'Concrete' manipulatives, concrete ideas. *Contemporary issues in early childhood*, *1*(1), 45-60.

Clements, D.H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, 6(2), 81–89.

Clements, D.H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the "Building Blocks" project. *Journal for Research in Mathematics Education*, 38(2), 136–163.

Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math* (3rd edition). New York: Routledge/Taylor & Francis.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development*, *13*(1), 6-33.

Committee on Defining Deeper Learning and 21st Century Skills. (2012). *Education for Life and Work: Developing Transferable Knowledge and Skills in the 21st Century*. J. Pellegrino & M. Hilton (Eds.). Board on Testing and Assessment and Board on Science Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Council of the Great City Schools. (2020). *Addressing Unfinished Learning After COVID-19 School Closures.* https://www.cgcs.org/cms/lib/DC00001581/Centricity/Domain/313/CGCS_Unfinished%20Learning.pdf

Cross, C.T., Woods, T.A., & Schweingruber, H. (Eds.). (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* Center for Education, Division of Behavioral and Social Science and Education. Washington, DC: National Academy Press.

Cuoco, A., Goldenberg, E.P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior*, 15(4), 375–402.

Czerniak, C.M., Weber, W.B., Jr., Sandmann, A., & Ahem, J. (1999). A literature review of science and mathematics integration. *School Science & Mathematics*, 99(8), 421–430.

Czupryk, B. (2020, April 30). Remediation won't help students catch up. Here's what will. *TNTP Blog*. https://tntp.org/blog/post/remediation-wont-help-students-catch-up-heres-what-will.

Dacey, L., Lynch, J. B., & Salemi, R. E. (2013). *How to differentiate your math instruction: lessons, ideas, and videos with common core support: a multimedia professional learning resource*. Math Solutions.

Darling-Hammond, L. (2010). *Performance counts: Assessment systems that support high-quality learning*. Washington, DC: Council of Chief State School Officers.

David, J. & Greene, D. (2007). *Improving Mathematics Instruction in Los Angeles High Schools: An Evaluation of the PRISMA Pilot Program*. Palo Alto, Calif.: Bay Area Research Group.

Delgado, A., L., Wardlow, K. McKnight, & K. O'Malley. (2015). Educational technology: A review of the integration, resources, and effectiveness of technology in K-12 classrooms. *Journal of Information Technology Education Research,* 14, 397-4.

Desoete, A., & De Craene, B. (2019). Metacognition and mathematics education: An overview. *ZDM*, *51*(4), 565-575.

Dewey, J. (1944). *Democracy and education*. New York: Macmillan. Retrieved August 2002, from http://www.ilt.columbia.edu/publications/dewey.html. [Originally published in 1916].

Dixon, J. (2018). Just-in-time vs. just-in-case scaffolding: How to foster productive perseverance. Retrieved online: https://www.hmhco.com/blog/just-in-time-vs-just-in-case-scaffolding-how-to-foster-productive-perseverance

Donovan, S., & Bransford, J. (2005). *How students learn*. National Research Council. Washington, DC: National Academies Press.

Dunn, K. E., Airola, D. T., Lo, W. J., & Garrison, M. (2013). Becoming data driven: The influence of teachers' sense of efficacy on concerns related to data-driven decision making. *The Journal of Experimental Education*, *81*(2), 222-241.

Dweck, C. (2006). *Mindset: The New Psychology of Success*. New York: Random House.

Dweck, C. (2008). *Mindsets and Math/Science Achievement*. New York: Carnegie Corporation of New York Institute for Advanced Study.

Dweck, C. (2015). Carol Dweck revisits the growth mindset. *Education Week*, *35*(5), 20-24.

Dyer, E. B., & Sherin, M. G. (2016). Instructional reasoning about interpretations of student thinking that supports responsive teaching in secondary mathematics. *ZDM*, *48*(1), 69-82.

Eccles, J.S., Wigfield, A., & Schiefele, U. (1998). Motivation to succeed. In *Handbook of child psychology: Volume 3 – Social, emotional, and personality development* (5th ed.). N. Eisenberg (Ed.). NY, NY: Wiley.

Farrington, C. A., Roderick, M., Allensworth, E., Nagaoka, J., Keyes, T. S., Johnson, D. W., & Beechum, N. O. (2012). *Teaching Adolescents to Become Learners: The Role of Noncognitive Factors in Shaping School Performance--A Critical Literature Review*. Chicago, IL: Consortium on Chicago School Research.

Feldman, K., & Kinsella, K. (2005). *Narrowing the language gap: The case for explicit vocabulary instruction*. NY, NY: Scholastic.

Fisher, D., Frey, N. & Hattie, J. (2020). *The distance learning playbook, grades K-12: Teaching for engagement and impact in any s*etting. Thousand Oaks, CA: Corwin.

Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap?. *The High School Journal*, *91*(1), 29-42.

Fonger, N., Stephens, A. Blanton, M., Isler, I. Knuth, E. & Murphy Gardiner, A. (2018). Developing a learning progression for curriculum, instruction, and student learning: An example from mathematics education. *Cognition and Instruction*, 36(1), 30-55.

Fosnot, C.T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.

Fosnot, C.T., & Jacob, W. (2010). *Young mathematicians at work: Constructing algebra*. Portsmouth, NH: Heinemann.

Fox, J. (2004). Test decisions over time: Tracking validity. *Language Testing*, 21, 437–465.

Francisco, J. M., & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *The Journal of Mathematical Behavior*, *24*(3-4), 361-372.

Franke, M. L., Kazemi, E., & Battey, D. S. (2007). Mathematics teaching and classroom practices. In F. K. Lester Jr. (Ed.), *The second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, NC: Information Age.

Fuchs, L.S. (2004). The past, present, and future of curriculum-based measurement research. *School Psychology Review*, 33, 188–192.

Fuchs, D., & Fuchs, L.S. (2006). Introduction to Response to Intervention: What, why, and how valid is it? *Reading Research Quarterly*,41(1), 93–99.

Fuson, K.C. (1986). Roles of representation and verbalization in the teaching of multidigit addition and subtraction. *European Journal of Psychology in Education*, 4, 35–56.

Fuson, K.C. (2009). Avoiding misinterpretations of Piaget and Vygotsky: Mathematical teaching without learning, learning without teaching, or helpful learning-path teaching? *Cognitive Development*, 24(4), 343–361.

Fuson, K.C., Kalchman, M., & Bransford, J.D. (2005). Mathematical understanding: An introduction. In M.S. Donovan & J.D. Bransford (Eds.), *How students learn: History, math, and science in the classroom* (pp. 217-256). Washington, DC: National Academy Press.

Fuson, K.C. & Murata, A. (2007). Integrating NRC principles and the NCTM Process Standards to form a Class Learning Path Model that individualizes within whole-class activities. National Council of Supervisors of Mathematics. *Journal of Mathematics Education Leadership,* 10(1), 72–91.

Gaddy, A.K., Harmon, S.E., Barlow, A.T., Milligan, C.D., & Huang, R. (2014). Implementing the Common Core: Applying shifts to instruction. *Mathematics Teacher*, 108(2), 108–113.

García, E., & Weiss, E. (2020). COVID-19 and Student Performance, Equity, and US Education Policy: Lessons from Pre-Pandemic Research to Inform Relief, Recovery, and Rebuilding. *Economic Policy Institute*.

Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, *38*(4), 915–945. https://doi.org/10.3102/00028312038004915

Garrison, D. & H. Kanuka. (2004). Blended learning: Uncovering its transformative potential in higher education. *The Internet and Higher Education*, 7(2), 95-105.

Gemin, B., & Pape, L. (2017). Keeping Pace with K-12 Online Learning, 2016. *Evergreen Education Group*.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J.R., & Witzel, B. (2009). Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools. Institute of Education Sciences What Works Clearinghouse. Washington, DC: U.S. Department of Education. Retrieved online: http://ies.ed.gov/ncee/wwc/pdf/practiceguides/rti_math_ pg_042109.pdf.

Gersten, R., & Chard, D. (2001). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *LD OnLine*. Retrieved online: http:// www.ldonline.org/article/5838/.

Gersten, R., Clarke, B., & Mazzocco, M. M. (2007). Historical and contemporary perspectives on mathematical learning disabilities. *Instruction Research Brief*. Reston, VA: National Council of Teachers of Mathematics.

Goffney, I. (2018). Where do we go from here? Next steps in rehumanizing mathematics for Black, Indigenous, and Latinx students. In R. Gutiérrez, I. Goffney, & M. Boston (Eds.), *Annual perspectives in mathematics. education 2018: Rehumanizing mathematics for Black, Indigenous, and Latinx students* (pp. 159–170). Reston, VA: National Council of Teachers of Mathematics.

Goldenberg, E., Mark, J., Kang, J., Fries, M., Carter, C., & Cordner, T. (2015). *Making sense of algebra: Developing students' mathematical habits of mind*. Portsmouth, NH: Heinemann.

Graham, C. R., Borup, J., Pulham, E., & Larsen, R. (2019). K–12 Blended teaching readiness: Model and instrument development. *Journal of Research on Technology in Education*, *51*(3), 239-258.

Graham, S., Kiuhara, S. A., & MacKay, M. (2020). The effects of writing on learning in science, social studies, and mathematics: A meta-analysis. *Review of Educational Research*, *90*(2), 179-226.

Granovskiy, B. (2018). Science, Technology, Engineering, and Mathematics (STEM) Education: An Overview. CRS Report R45223, Version 4. Updated. *Congressional Research Service*.

Grapin, S. (2019). Multimodality in the new content standards era: Implications for English learners. *Tesol Quarterly*, *53*(1), 30-55.

Gulamhussein, A. (2013). *Teaching the teachers: Effective professional development in an era of high stakes accountability*. Alexandria, VA: Center for Public Education.

Guthrie, J. T., & Humenick, N. M. (2004). Motivating Students to Read: Evidence for Classroom Practices that Increase Reading Motivation and Achievement. In P. McCardle & V. Chhabra (Eds.), *The voice of evidence in reading research* (pp. 329–354). Paul H Brookes Publishing Co.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education* 44, (1), 37–68.

Harlen, W. (2005). Teachers' summative practices and assessment for learning—tensions and synergies. *The Curriculum Journal, 16*(2), 207–223.

Hatano, G., & Inagaki, K. (1991). Constrained person analogy in young children's biological inference. *Cognitive Development*, 6(2), 219–231.

Hammer, D., Goldberg, F., & Fargason, S. (2012). Responsive teaching and the beginnings of energy in a third grade classroom. *Review of science, mathematics and ICT education*, *6*(1), 51-72.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta- analyses relating to achievement*. London, UK: Routledge.

Hattie, J., & Clarke, S. (2018). *Visible Learning: Feedback*. Routledge.

Hattie, J., Fisher, D. & Frey, N. (2017). *Visible Learning for Mathematics: What Works Best to Optimize Student Learning, Grades K-12*. Thousand Oaks, CA: Corwin Mathematics.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81–112.

Haystead, M. & Marzano, R. (2009). Meta-analytic synthesis of studies conducted at Marzano Research Laboratory on instructional strategies. *Englewood, CO: Marzano Research Laboratory*.

Heineke, S., & B. Polnick. (2013). Pave the way for coaches. *Journal of Staff Development*, *34*(3), 48–51.

Henderson, L., J. Klemes & Y. Eshet. (2000). Just playing a game? Educational simulation software and cognitive outcomes. *Journal of Educational Computing Research,* 22(1), 105-129.

Heritage, M. (2007). Formative assessment: What do teachers need to know and do? *Phi Delta Kappan*, 89(2), 140–145.

Heritage, M. (2008). Learning progressions: Supporting instruction and formative assessment. The Council of Chief State School Officers. Retrieved online: http://169.62.82.226/documents/mde/CCSSO_Learning_Progressions_Mararget_Heritage_1_601110_7.pdf

Hibbard, M. (1996). *A teacher's guide to performance-based learning and assessment*. Alexandria, VA: ASCD.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1), 3–19.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researche*r, 25(4), 12–21.

Hiebert, J., & Grouws, D.A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on the teaching and learning of mathematics*. Reston, VA: National Council of Teachers of Mathematics. 371–404.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of teacher education*, *58*(1), 47-61.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal*, *30*(2), 393-425.

Hill, M. Sharma, M. O'Byrne, J. & Airey, J. (2014). Developing and evaluating a survey for representational fluency in science. *International Journal of Innovation in Mathematics and Science Education*, 22(6). Retrieved online: https://openjournals.library.sydney.edu.au/index.php/CAL/article/view/7484

Horn, M. & H. Staker. (2011). *The Rise of K-12 Blended Learning*. Lexington, MA: Innosight Institute.

Hufferd-Ackles, K., Fuson, K.C., & Sherin, M.G. (2004). Describing levels and components of a Math-Talk Learning Community. *Journal for Research in Mathematics Education*, 35(2), 81–116.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2015). Describing levels and components of a Math-Talk Learning Community. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices*, (pp. 125-134). Reston, VA: NCTM.

Jackson, R. R., & Lambert, C. (2010). *How to Support Struggling Students. Mastering the Principles of Great Teaching series*. ASCD. 1703 North Beauregard Street, Alexandria, VA 22311-1714.

Jackson, C., Mohr-Schroeder, M. J., Bush, S. B., Maiorca, C., Roberts, T., Yost, C., & Fowler, A. (2021). Equity-oriented conceptual framework for K-12 STEM literacy. *International Journal of STEM Education*, *8*(1), 1-16.

Jacobs, V. & Ambrose, R. (2008). Making the most of story problems. *Teaching children mathematics*, *15*(5), 260-266.

Jacobs, V., Lamb, L. & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education* 41(2),169–202.

Janzen, J. (2008). Teaching English language learners in the content areas. *review of Educational research*, *78*(4), 1010-1038.

Joyce, B., & Showers, B. (1982). The coaching of teaching. *Educational Leadership*, *40*(1), 4–8.

Kalbfleisch, M. L., & Tomlinson, C. A. (1998). Teach me, teach my brain a call for differentiated classrooms. *Educational Leadership*, *56*(3), 52-55.

Kang, H., Calabrese Barton, A., Tan, E., D Simpkins, S., Rhee, H. Y., & Turner, C. (2019). How do middle school girls of color develop STEM identities? Middle school girls' participation in science activities and identification with STEM careers. *Science Education*, *103*(2), 418-439.

Kanold, T. (2018). *Mathematics RTI: A high quality response when students don't learn!* HMH Driving Student Outcomes with Intentional Instruction Summit.

Kaplinsky, R. (2018). Mathematical modeling: Do you need better spies or analysts? Retrieved online: https://robertkaplinsky.com/mathematical-modeling-need-better-spies-analysts/.

Kaplinsky, R. (2019). Getting to the heart of what students know in math. *Edutopia.* Retrieved online: https://www.edutopia.org/article/getting-heart-what-students-know-math.

Kapur, M. (2010). Productive failure in mathematical problem solving. *Instructional Science*, 38(6), 523–550.

Kapur, M. (2014). Productive failure in learning math. *Cognitive science*, *38*(5), 1008-1022.

Kelemanik, G., Lucenta, A. & Creighton, S. (2016). *Routines for Reasoning: Fostering the Mathematical Practices in All Students*. Portsmouth, NH: Heinemann.

Ketterlin-Geller, L.R., & Yovanoff, P. (2009). Diagnostic assessments in mathematics to support instructional decision making. *Practical Assessment, Research & Evaluation*, 14(16), 1–1.

Kieschnick, W. (2017). *Bold school: Old school wisdom + new school technologies*. Highbridge Audio.

Kingston, N., & Nash, B. (2011). Formative assessment: A meta-analysis and a call for research. Educational Measurement: Issues and Practice, 30(4), 28–37.

Kisker, E. E., Lipka, J., Adams, B. L., Rickard, A., Andrew-Ihrke, D., Yanez, E. E., & Millard, A. (2012). The potential of a culturally based supplemental mathematics curriculum to improve the mathematics performance of Alaska Native and other students. *Journal for Research in Mathematics Education*, *43*(1), 75-113.

Klibanoff, R.S., Levine, S.C., Huttenlocher, J., Vasilyeva, M., & Hedges, L.V. (2006). *Preschool children's mathematical* knowledge: The effect of teacher 'math talk.' *Developmental Psychology*, 42(1), 59–69.

Klute, M., Apthorp, H., Harlacher, J., & Reale, M. (2017). Formative Assessment and Elementary School Student Academic Achievement: A Review of the Evidence. REL 2017-259. *Regional Educational Laboratory Central*.

Knight, J. (2011). *Unmistakable impact: A partnership approach for dramatically improving instruction*. Thousand Oaks, CA: Corwin Press.

Kovalik, S., & Olsen, K. (1994). *ITI: The model. Integrated thematic instruction*. Kent, WA: Books for Educators.

Kraft, M., Blazar, D. & Hogan, D. (2018). The effect of teaching coaching on instruction and achievement: A meta-analysis of the causal evidence. *Review of Educational Research, 88*(4), 547–588.

Kuhfeld, M., Tarasawa, B., Johnson, A., Ruzek, E., & Lewis, K. (2020). Learning during COVID-19: Initial findings on students' reading and math achievement and growth. NWEA Brief: Portland, OR.

Kwon, J. B., DeBruler, K., & Kennedy, K. (2019). A snapshot of successful K-12 online learning: Focused on the 2015-16 academic year in Michigan. *Journal of Online Learning Research*, *5*(2), 199-225.

Lampert, M. (2015). *Deeper teaching. Students at the center: Deeper learning research series.* Boston, MA: Jobs for the Future.

Lane, S. (2013). Performance assessment. In J.H. McMillan (Ed.), *SAGE handbook of research on classroom assessment* (pp. 313-329). Thousand Oaks, CA: Sage.

Langdon, D., McKittrick, G., Beede, D., Khan, B., & Doms, M. (2011). STEM: Good Jobs Now and for the Future. ESA Issue Brief# 03-11. Washington, DC: US Department of Commerce.

Larson, M. (2016). The need to make homework comprehensible. Retrieved online: https://www.nctm.org/News-and-Calendar/Messages-from-thePresident/Archive/Matt-Larson/The-Need-to-Make-Homework-Comprehensible.

Larson, M. (2018). Equity is more than access. *Math Solutions*. Retrieved online: https://mathsolutions.com/uncategorized/equity-is-more-than-access/.

Larson, M. (2017). Mathematics Learning: A Journey Not a Sprint. Retrieved online: https://my.nctm.org/blogs/matthew-larson/2017/12/20/mathematics-learning-a-journey-not-a-sprint.

Larson, M. R. & Kanold, T. D. (2016). *Balancing the Equation: A Guide to School Mathematics for Educators and Parents: Contexts for Effective Student Learning in the Common Core).* Bloomington, IN: Solution Tree Press.

Lawrence-Brown, D. (2004). Differentiated instruction: Inclusive strategies for standards-based learning that benefit the whole class. *American secondary education*, 34-62.

Leahy, S., Lyon, C., Thompson, M. & Wiliam, D. (2005). Classroom assessment: Minute by minute, day by day. *Educational Leadership,* 63(3), 18–24.

Lee, H., Chung, H. Q., Zhang, Y., Abedi, J., & Warschauer, M. (2020). The effectiveness and features of formative assessment in US K-12 education: A systematic review. *Applied Measurement in Education*, *33*(2), 124-140.

Leinwand, S., & Fleischman, S. (2004). Teach mathematics right the first time. *Educational Leadership*, 62(1), 88–89.

Lembke, E., & Foegen, A. (2005). *Identifying indicators of early mathematics proficiency in Kindergarten and Grade 1.* Technical Report 6). Minneapolis, MN: University of Minnesota, College of Education and Human Development, Research Institute on Progress Monitoring.

Levasseur, K. & Cuoco, A. (2009). Mathematical Habits of Mind. In *Teaching Mathematics through Problem Solving*, pp. 34-35. NCTM: Reston, VA.

Lipka, J., Sharp, N., Adams, B., & Sharp, F. (2007). Creating a third space for authentic biculturalism: Examples from math in a cultural context. *Journal of American Indian Education*, 94-115.

Lucretia, A. & Kelemanik, G. (2020). A Routine for Reasoning to Ensure ALL Students are Modeling with Mathematics. NCTM Professional Development Webinars. Retrieved online: https://www.nctm.org/online-learning/Webinars/Details/495

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. 2nd ed. NY, NY: Routledge.

Maher, C. A. (2002). How Students Structure Their Own Investigations and Educate Us: What We've Learned from a Fourteen Year Study. Journal of Mathematical Behavior, 24, 1-14.

Marcus, R., & Fey, J. T. (2003). Selecting quality tasks for problem-based teaching. In H. L. Schoen (Ed.), *Teaching mathematics through problem solving: Grades 6–12* (pp. 55–67). Reston, VA: National Council of Teachers of Mathematics.

Marsh, J. A., Pane, J. F., & Hamilton, L. S. (2006). Making Sense of Data-Driven Decision Making in Education: Evidence from Recent RAND Research. Occasional Paper. Rand Corporation.

Masingila, J. O., Olanoff, D., & Kimani, P. M. (2018). Mathematical knowledge for teaching teachers: Knowledge used and developed by mathematics teacher educators in learning to teach via problem solving. *Journal of Mathematics Teacher Education*, *21*(5), 429-450.

Matsumura, L.C., Correnti, R., Walsh, M., DiPrima Bickel, D., & Zook-Howell, D. (2019) Online content-focused coaching to improve classroom discussion quality, *Technology, Pedagogy and Education, 28*(2), 191–215.

Mayer, R. (2013). Multimedia learning. In *Educational Psychology Handbook: International Guide to Student Achievement,* J. Hattie & E. Anderman (Eds.). 396-398. New York: Routledge.

Mayes, R., Chase, P. N., & Walker, V. L. (2008). Supplemental practice and diagnostic assessment in an applied college algebra course. *Journal of College Reading and Learning*, *38*(2), 7-30.

McKenzie, K. B., Skrla, L., Scheurich, J. J., Rice, D., & Hawes, D. P. (2011). Math and science academic success in three large, diverse, urban high schools: a teachers' story. *Journal of Education for Students Placed at Risk*, *16*(2), 100-121.

McLeskey, J., Waldron, N. L., So, T. S. H., Swanson, K., & Loveland, T. (2001). Perspectives of teachers toward inclusive school programs. *Teacher education and special education*, *24*(2), 108-115.

McTighe, J., & Wiggins, G. (2013). *Essential questions: Opening doors to student understanding*. Alexandria, VA: ASCD.

Meeuwse, K. & Mason, D. (2018). *Personalized professional learning for educators: Emerging research and opportunities.* Hershey, PA: IGI Global.

Mercer, N., & Howe, C. (2012). Explaining the dialogic processes of teaching and learning: The value and potential of sociocultural theory. *Learning, culture and social interaction*, *1*(1), 12-21.

Merchant, Z., E. Goetz, W. Kenney-Kennicutt, O. Kwok, L. Cifuentes & T. Davis. (2012). The learner characteristics, features of desktop 3D virtual reality environments, and college chemistry instruction: A structural equation modeling analysis. *Computers & Education*, 59(2), 551-568.

Michaels, S., O'Connor, C., & Resnick, L. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education*, 27(4), 283–297.

Mid-Continent Research for Education and Learning (McREL). (2010). *What we know about mathematics teaching and learning, third edition*. Bloomington, IN: Solution Tree Press.

Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice*, *22*(1), 47-57.

Miri, B., David, B-C., & Uri, Z. (2007). Purposely teaching for the promotion of higher-order thinking skills: A case of critical thinking. *Research in Science Education*, 37(4), 353–369.

Molnar, M. (2014). Richard Culatta: Five ways technology can close equity gaps. *Education Week* (*Ed Week Market Brief*). Retrieved online: https://marketbrief.edweek.org/marketplace-k-12/richard_culatta_five_ways_technology_can_close_equity_gaps.html

Morgan, C., Craig, T., Schütte, M. & Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. *ZDM: The International Journal of Mathematics Education*, 46 (6), 843-853.

Moyer, P. S., & Milewicz, E. (2002). Learning to question: Categories of questioning used by preservice teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education*, *5*(4), 293-315.

Mueller, M., Yankelewitz, D., & Maher, C. (2014). Teachers promoting student mathematical reasoning. *Investigations in Mathematics Learning*, *7*(2), 1-20.

National Center for Education Statistics. (2016). *The condition of education 2016*. Washington, DC: U.S. Department of Education.

National Council of Teachers of Mathematics. (2000, 2009). *Principles and standards for school mathematics*. Reston, VA: Author. Retrieved online: http://www.nctm.org/ standards/content.aspx?id=16909.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2015). *Position statement on procedural fluency in mathematics*. Reston, VA: Author. Retrieved online: http://www.nctm.org/ Standards-and-Positions/Position-Statements/Procedural- Fluency-in-Mathematics/

National Council of Teachers of Mathematics. (2016). NCTM Position Statement: Large-Scale Mathematics Assessment and High-Stakes Decisions. Retrieved online: https://www.nctm.org/uploadedFiles/Standards_and_Positions/Position_Statements/Large-Scale%20Assessments%200816.pdf

National Council of Teachers of Mathematics. (2022). NCTM Position Statement: https://www.nctm.org/Standards-and-Positions/Position-Statements/Access-and-Equity-in-Mathematics-Education/*Moving Forward: Mathematics Learning in the Era of COVID-19*.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education. Retrieved online: http://www2.ed.gov/about/ bdscomm/list/mathpanel/index.html.

National Research Council. (1999). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.).
Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Science and Education. Washington, DC: National Academy Press.

National Research Council. (2005). *How students learn: Mathematics in the classroom*. Washington, DC: National Academies Press. Retrieved online: from http://www. nap.edu/openbook.php?isbn=0309089492

National Research Council. (2012). *Education for life and work: Developing transferable knowledge and skills for the 21st century*. Washington, DC: National Academies Press.

Osguthorpe, R. & C. Graham. (2003). Blended learning systems: Definitions and directions. *Quarterly Review of Distance Education,* 4(3), 227–234.

Page, S., & Clarke, J. (2014). Feeling your way to success through journaling. *Australian Primary Mathematics Classroom*, *19*(1), 3-8.

Pak, K., Polikoff, M. S., Desimone, L. M., & Saldívar García, E. (2020). The adaptive challenges of curriculum implementation: Insights for educational leaders driving standards-based reform. *AERA Open*, *6*(2), 2332858420932828.

Patrick, S. & A. Powell. (2009). *A Summary of Research on the Effectiveness of K-12 Online Learning*. Vienna, VA: International Association for K-12 Online Learning.

Polya, G. (1965). Mathematical Discovery, Volume II: On Understanding, Learning, and Teaching Problem Solving. New York, NY: Wiley.

Popham, W. J. (2008). *Transformative assessment*. Washington DC: Association for Supervision and Curriculum Development.

Powell, S. R., Hebert, M. A., Cohen, J. A., Casa, T. M., & Firmender, J. M. (2017). A Synthesis of Mathematics Writing: Assessments, Interventions, and Surveys. *Journal of Writing Research*, *8*(3).

Presmeg, N. (2020). Visualization and learning in mathematics education. *Encyclopedia of mathematics education*, 900-904.

Public Impact. (2013). *A Better Blend: A Vision for Boosting Student Outcomes with Digital Learning*. Chapel Hill, NC: Author.

Raley, S. K., Shogren, K. A., & McDonald, A. (2018). How to implement the self-determined learning model of instruction in inclusive general education classrooms. *Teaching Exceptional Children*, *51*(1), 62-71.

Riccomini, P. J., Witzel, B. S., & Deshpande, D. S. (2022). Combining Visual Representations and a Powerful Retention Strategy With Peer-Mediated Strategies to Improve Mathematical Outcomes for Students With EBD. *Beyond Behavior*, *31*(1), 42-52.

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives*, *11*(3), 184-190.

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review*, *27*(4), 587-597.

Rock, M.L. (2019). *The eCoaching continuum for educators: Using technology to enrich professional development and improve student outcomes.* Alexandria, VA: ASCD.

Rohrer, D. (2009). The effects of spacing and mixed practice problems. *Journal for Research in Mathematics Education*, 40(1), 4–17.

Ross, S., G. Morrison & D. Lowther. (2010). Educational technology research past and present: Balancing rigor and relevance to impact school learning. *Contemporary Educational Technology*, 1(1), 17-35.

Russek, B. (1998). Writing to learn mathematics. *Writing Across the Curriculum*, *9*, 36-45.

Russell, S.J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics*, 7(3), 154–158.

Russo, M., Hecht, D., Burghardt, M.D., Hacker, M., & Saxman, L. (2011). Development of a multidisciplinary middle school mathematics infusion model. *Middle Grades Research Journal*, 6 (2), 113–128.

Santos-Trigo M. (2020) Problem-Solving in Mathematics Education. In: Lerman S. (eds) Encyclopedia of Mathematics Education. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_129

Sarama, J., DiBiase, A. M., Clements, D. H., & Spitler, M. E. (2004). The professional development challenge in preschool mathematics. *Engaging young children in mathematics: Standards for early childhood mathematics education*, 415-446.

Saxe, G. B., Gearhart, M., & Nasir, N. I. S. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of mathematics teacher education*, *4*(1), 55-79.

Saylor, L. L., & Walton, J. B. (2018). Creating a math-talk learning community with preservice teachers. *School Science and Mathematics*, *118*(8), 348-357.

Schifter, C. C., Natarajan, U., Ketelhut, D. J., & Kirchgessner, A. (2014). Data-Driven Decision Making: Facilitating Teacher Use of Student Data to Inform Classroom Instruction. *Contemporary Issues in Technology and Teacher Education*, *14*(4), 419-432.

Schmidt, W. H., Wang, H. C., & McKnight, C. C. (2005). Curriculum coherence: An examination of US mathematics and science content standards from an international perspective. *Journal of curriculum studies*, *37*(5), 525-559.

Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. *ZDM*, *42*(2), 149-161.

Schneider, M.C., Egan, K.L., & Julian, M.W. (2013). Classroom assessment in the context of high-stakes testing. In J.H. McMillan (Ed.), *SAGE handbook of research on classroom assessment* (pp. 55–70). Thousand Oaks, CA: Sage.

Schunk, D., P. Pintrich, & J. Meece. (2008). *Motivation in education: Theory, research, and applications.* Upper Saddle River, New Jersey: Pearson/Merrill Prentice Hall.

Senn, D., Rutherford, A. C., & Marzano, R. J. (2014). *Identifying critical content: Classroom techniques to help students know what is important*. West Palm Beach, FL: Learning Sciences International.

Shannon, G.S., & Bylsma, P. (2003). *Nine characteristics of high-performing schools: A research-based resource for school leadership teams to assists with the School Improvement Process*. Olympia, Washington: Office of the School Superintendent of Public Instruction.

Shapka, J. D., Domene, J. F., & Keating, D. P. (2006). Trajectories of career aspirations through adolescence and young adulthood: Early math achievement as a critical filter. *Educational Research and Evaluation*, *12*(4), 347-358.

Sherin, B., & Fuson, K. (2005). Multiplication strategies and the appropriation of computational resources. *Journal for Research in Mathematics Education*, 36(4), 347–395.

Sherin, M. & van Es, E. (2003). A new lens on teaching: Learning to notice. *Mathematics teaching in the middle school*, *9*(2), 92-95.

Sims, R., G. Dobbs & T. Hand. (2002). Enhancing quality in online learning: Scaffolding planning and design through proactive evaluation. *Distance Education*, 23(2), 135–148.

Sircar, S., & Titus, S. (2015). Keeping things in proportion. *At Right Angles*, *4*(2), 30-35.

Smith, M. & Stein, M. (2011). 5 Practices for orchestrating productive mathematics discussions. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Smith, M.S., & Stein, M.K. (2018). *Five practices for orchestrating productive mathematics discussions* (2nd ed). Resnick, VA: NCTM.

Sneider, C. I., & Ravel, M. K. (2021). Insights from Two Decades of P-12 Engineering Education Research. *Journal of Pre-College Engineering Education Research (J-PEER)*, *11*(2), 5.

Star, J. R. (2015). When not to persevere – Nuances related to perseverance in mathematical problem solving. Chicago, IL: Spencer Foundation. Retrieved from http://hub.mspnet.org/index.cfm/28127/

Stein, M.K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, 2(1), 50–80.

Steiner, D., Weisberg, D, (2020). When Students Go Back to School, Too Many Will Start the Year Behind. Here's How to Catch Them Up — in Real Time. *The 74 Million.* (2020, April 26).

Stetson, R., Stetson, E., & Anderson, K. A. (2007). Differentiated instruction, from teachers' experiences. *The School Administrator*, 8 (64), online. Retrieved online: http://www.aasa.org/SchoolAdministratorArticle. aspx?id=6528.

Stewart, J., Cartier, J. L., & Passmore, C. M. (2005). Developing understanding through model-based inquiry. In M.S. Donovan & J.D. Bransford, (Eds), *How How students learn: Science in the classroom (pp.* 515-565). National Research Council, Committee on *How People Learn*, A Targeted Report for Teachers, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Stiles, J. (2016). Supporting Mathematical Discourse in the Early Grades. Interactive STEM Research+ Practice Brief. *Education Development Center, Inc.*

Strangman, N., Hall, T., & Meyer, A. (2004). Background knowledge instruction and the implications for UDL implementation. *Retrieved Oct*, *23*, 2006.

Stylianou, D. A. (2011). An examination of middle school students' representation practices in mathematical problem solving through the lens of expert work: Towards an organizing scheme. *Educational Studies in Mathematics, 76,* 265-280.

Stylianou, D.A., & Silver, E.A. (2004). The role of visual representations in advanced mathematical problem solving: An examination of expert-novice similarities and differences. *Mathematical Thinking and Learning*, 6(4), 353–387.

Sweeney, D. (2011). *Student-centered coaching.* Thousand Oaks, CA: Corwin Press.

Sztajn, P., Confrey, J., Wilson, P.H., & Edgington, C. (2012). Learning trajectory-based instruction: Towards a theory of teaching. *Educational Researcher*, 41(5), 147–156.

Tamim, R., R. Bernard, E. Borokhovski, P. Abrami & R. Schmid. (2011). What forty years of research says about the impact of technology on learning: A second-order meta-analysis and validation study. *Review of Educational Research*, 81(1), 4-28.

Tarasawa, B. & Samuel, A. (2021, January 13). Learning during COVID-19: Initial findings and 4 considerations for policymakers. *Education Commission of the States – Ed Note Blog*. https://ednote.ecs.org/learning-during-covid-19-initial-findings-and-4-considerations-for-policymakers/

Tarr, J. E., Reys, R. E., Reys, B. J., Chávez, Ó., Shih, J., & Osterlind, S. J. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for research in mathematics education*, *39*(3), 247-280.

Taylor, R., & Chanter, C. (2016). *The Coaching partnership: Tips for improving coach, mentor, teacher, and administrator effectiveness.* Lanham, MD: Rowman & Littlefield.

Taylor, J. A., & McDonald, C. (2007). Writing in groups as a tool for non-routine problem solving in first year university mathematics. *International Journal of Mathematical Education in Science and Technology*, *38*(5), 639-655.

Taylor, L. & J. Parsons. (2011). Improving student engagement. *Current Issues in Education*, 14(1). Retrieved from: http://cie.asu.edu/.

Tibbitt, J. (2020). Formative Assessment: A Tool for Closing Achievement Gaps in Diverse Classrooms. *Odyssey: New Directions in Deaf Education*, *21*, 72-75.

Tomlinson, C.A. (1997). Meeting the needs of gifted learners in the regular classroom: Vision or delusion? *Tempo*, 17(1), 1, 10–12

Tomlinson, C. A. (2001). *How to differentiate instruction in mixed-ability classrooms*. Alexandria, VA: ASCD.

Tomlinson, C.A., (2005). Traveling the road to differentiation in staff development. *Journal of Staff Development*, 26, 8–12.

Tomlinson, C. A., & Allan, S. D. (2000). *Leadership for differentiating schools and classrooms*. Alexandria, VA: ASCD.

Tucker, B. (2012). The flipped classroom. *Education Next*, 12(1), 82-83.

Uğur, B., B. Akkoyunlu & S. Kurbanoğlu. (2011). Students' opinions on blended learning and its implementation styles. *Education and Information Tech*nologies, 16(1), 5-23.

Ukpokodu, O. N. (2011). How do I teach mathematics in a culturally responsive way?: Identifying empowering teaching practices. *Multicultural Education*, *19*(3), 47-56.

U.S. Department of Education, Office of Educational Technology. (2016). *Future Ready Learning: Reimagining the Role of Technology in Education*. Washington, DC: Author.

U.S. Department of Education, Office of Educational Technology. (2019). *Nine Dimensions for Supporting STEM Learning with Technology*. Washington, DC: Author.

U.S. Department of Education, Office of English Language Acquisition. (2020.) *Integration of English Language while Teaching Mathematics*, Washington, DC: Author.

Urquhart, V. (2009). Using Writing to Improve Math Learning. *Middle Ground*, *12*(4), 17.

Van Garderen, D. (2006). Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. *Journal of learning disabilities*, *39*(6), 496-506.

Warshauer, H. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*, 18(4), 375-400.

Watson, J. M., Campbell, K. J., & Collis, K. F. (1996). Fairness and fractions in early childhood. In *Technology in Mathematics Education* (1), 588-595.

Wei, R., Darling-Hammond, L., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher professional development in the United States and abroad* (technical report). Washington, DC: National Staff Development Council.

WIDA. (2020). *WIDA English language development standards framework, 2020 edition. Kindergarten-grade 12*. Board of Regents of the University of Wisconsin System.

Wiggins, G., & McTighe, J. (2005). *Understanding by design*. Alexandria, VA: Association for Supervision and Curriculum Development.

Wilcox, B., & Monroe, E. E. (2011). Integrating writing and mathematics. *The Reading Teacher*, *64*(7), 521-529.

Wiliam, D. (2010). An integrative summary of the research literature and implications for a new theory of formative assessment. In H. Andrade & G. Cizek (Eds.), The handbook of formative assessment (pp. 17–40). New York, NY: Routledge.

Wiliam, D. (2011). *Embedded Formative Assessment*. Bloomington, IN: Solution Tree Press.

Wiliam, D. (2018). Assessment for learning: meeting the challenge of implementation. *Assessment in Education: Principles, Policy & Practice*, 25(6), 682-685.

Williams, K. M. (2003). Writing about the problem solving process to improve problem solving performance. *The Mathematics Teacher*, *96*(3), 185-187.

Wood, K., Kissel, B., & Haag, K. (2014). *What happens after staff development: A model for self-coaching in literacy.* Newark, DE: International Reading Association.

Xenofontos, C. (2019). Equity and social justice in mathematics education: A brief introduction. In C. Xenofontos (Ed.), *Equity in Mathematics Education: Addressing a Changing World*. Charlotte, NC: Information Age Publishing, Inc. pp. 1-23.

Yang, D. C., & Sianturi, I. A. J. (2019). Assessing students' conceptual understanding using an online three-tier diagnostic test. *Journal of Computer Assisted Learning*, *35*(5), 678-689.

Yeager, D., Walton, G., & Cohen, G. L. (2013). Addressing achievement gaps with psychological interventions. *Phi Delta Kappan*, *94*(5), 62-65.

Yoder, N. (2014). Teaching the Whole Child: Instructional Practices That Support Social-Emotional Learning in Three Teacher Evaluation Frameworks. Research-to-Practice Brief. *Center on Great Teachers and Leaders*.

Yoon, K., Duncan, T., Lee, T., Scarloss, B., & Shapley, K. (2007). *Reviewing the evidence on how teacher professional development affects student achievement* (Issues & Answers Report, REL 2007-No. 033). San Antonio, TX: Regional Educational Laboratory Southwest.

Yow, J. A. (2015). "Can You Tell Me More?" Student Journaling and Reasoning. *Mathematics Teaching in the Middle School*, *21*(2), 72-76.

Zhang, D. (2005). Interactive multimedia-based e-learning: A study of effectiveness. *The American Journal of Distance Education*, 19(3), 149-162.

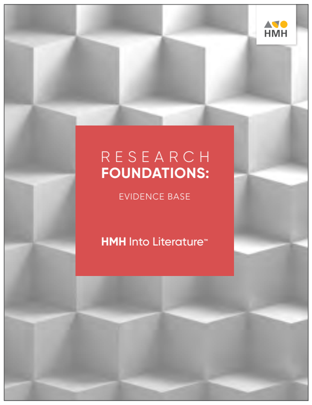
Zimmerman, R. H., Maker, C. J., & Alfaiz, F. (2020). Culturally responsive assessment of life science skills and abilities: Development, field testing, implementation, and results. *Journal of Advanced Academics*, *31*(3), 329-366.

Zwiers, J. (2014). *Building academic language: Meeting common core standards across disciplines, grades 5-12*. John Wiley & Sons.

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the design of mathematics curricula: Promoting language and content development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

HMH RESEARCH PUBLICATIONS

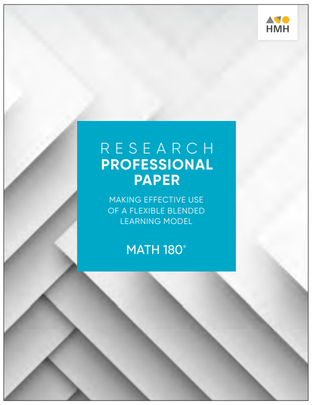
Research Into Practice Into Results



RESEARCH
FOUNDATIONS:
EVIDENCE BASE
HMH Into Literature™

RESEARCH FOUNDATIONS PAPERS

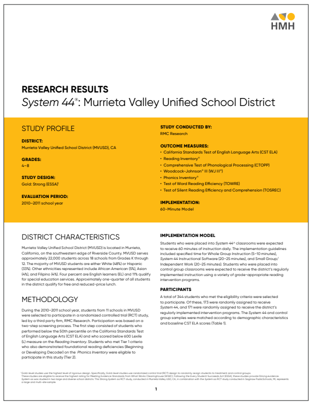
Research Foundations papers provide an in-depth account of the theoretical underpinnings, evidence base, and expert opinions that guide the design and development of new and revised programs. These papers map known research and design principles to practical applications of the program.



RESEARCH
PROFESSIONAL
PAPER
MAKING EFFECTIVE USE
OF A FLEXIBLE BLENDED
LEARNING MODEL
MATH 180®

RESEARCH PROFESSIONAL PAPERS

Research Professional Papers highlight an important theoretical construct, practical application, program component, or other topic related to learning in the context of HMH programs. They are authored by experts in the field, researchers, and thought leaders within the industry.



RESEARCH RESULTS
System 44®: Murrieta Valley Unified School District

STUDY PROFILE

DISTRICT:
Murrieta Valley Unified School District (MVUSD), CA

GRADES:
4–8

STUDY DESIGN:
Gold: Strong (ESSA)¹

EVALUATION PERIOD:
2010–2011 school year

STUDY CONDUCTED BY:
RMC Research

OUTCOME MEASURES:
- California Standards Test of English Language Arts (CST ELA)
- Reading Inventory®
- Comprehensive Test of Phonological Processing (CTOPP)
- Woodcock-Johnson® III (WJ III®)
- Phonics Inventory®
- Test of Word Reading Efficiency (TOWRE)
- Test of Silent Reading Efficiency and Comprehension (TOSREC)

IMPLEMENTATION:
60-Minute Model

DISTRICT CHARACTERISTICS

Murrieta Valley Unified School District (MVUSD) is located in Murrieta, California, on the southwestern edge of Riverside County. MVUSD serves approximately 22,000 students across 18 schools from Grades K through 12. The majority of MVUSD students are either White (48%) or Hispanic (33%). Other ethnicities represented include African American (5%), Asian (4%), and Filipino (4%). Four percent are English learners (EL) and 11% qualify for special education services. Approximately one-quarter of all students in the district qualify for free and reduced-price lunch.

METHODOLOGY

During the 2010–2011 school year, students from 11 schools in MVUSD were selected to participate in a randomized controlled trial (RCT) study, led by a third party firm, RMC Research. Participation was based on a two-step screening process. The first step consisted of students who performed below the 50th percentile on the California Standards Test of English Language Arts (CST ELA) and who scored below 600 Lexile (L) measure on the *Reading Inventory*. Students who met Tier 1 criteria who also demonstrated foundational reading deficiencies (Beginning or Developing Decoder) on the *Phonics Inventory* were eligible to participate in this study (Tier 2).

IMPLEMENTATION MODEL

Students who were placed into *System 44*® classrooms were expected to receive 60 minutes of instruction daily. The implementation guidelines included specified time for Whole Group Instruction (5–10 minutes), *System 44* Instructional Software (20–25 minutes), and Small Group/Independent Work (20–25 minutes). Students who were placed into control group classrooms were expected to receive the district's regularly implemented instruction using a variety of grade-appropriate reading intervention programs.

PARTICIPANTS

A total of 344 students who met the eligibility criteria were selected to participate. Of these, 173 were randomly assigned to receive *System 44*, and 171 were randomly assigned to receive the district's regularly implemented intervention programs. The *System 44* and control group samples were matched according to demographic characteristics and baseline CST ELA scores (Table 1).

[1] Gold-level studies use the highest level of rigorous design. Specifically, Gold-level studies use randomized control trial (RCT) design to randomly assign students to treatment and control groups. These studies are eligible to receive the highest rating for Meeting Evidence Standards from What Works Clearinghouse (WWC). Following the Every Student Succeeds Act (ESSA), these studies provide Strong evidence. *System 44* was studied in two large and diverse school districts. This Strong *System 44* RCT study, conducted in Murrieta Valley USD, CA, in combination with the *System 44* RCT study conducted in Saginaw PublicSchools, MI, represents a large and multi-site sample.

1

RESEARCH RESULTS PAPERS

Research Results papers summarize the findings from research studies conducted on HMH programs, including research conducted internally by HMH and externally by third-party research firms. Research Results papers document the efficacy of a program in terms of ESSA evidence levels: strong evidence, moderate evidence, promising evidence, and evidence that demonstrates a rationale for program effectiveness.

To learn more about HMH's dedication to research and efficacy, visit

hmhco.com/research

HMH Into Math

RESEARCH
EVIDENCE BASED

Browse our library of research at **hmhco.com/researchlibrary**

Houghton Mifflin Harcourt.

hmhco.com